Credit Cycles

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Abstract

This paper is a theoretical study into how credit constraints interact with aggregate economic activity over the business cycle. We construct a model of a dynamic economy in which lenders cannot force borrowers to repay their debts unless the debts are secured. In such an economy, durable assets such as land, buildings and machinery play a dual role: they are not only factors of production, but they also serve as collateral for loans. Borrowers' credit limits are affected by the prices of the collateralized assets. And at the same time, these prices are affected by the size of the credit limits. The dynamic interaction between credit limits and asset prices turns out to be a powerful transmission mechanism by which the effects of shocks persist, amplify, and spill over to other sectors. We show that small, temporary shocks to technology or income distribution can generate large, persistent fluctuations in output and asset prices.

1. Introduction

This paper is a theoretical study into how credit constraints interact with aggregate economic activity over the business cycle. In particular, for an economy where credit limits are endogenously determined, we investigate how relatively small, temporary shocks to technology or income distribution might generate large, persistent fluctuations in output and asset prices. Also we ask whether sector-specific shocks can be contagious, in the sense that their effects spill over to other sectors and get amplified through time.

For this purpose, we construct a model of a dynamic economy in which credit constraints arise naturally, due to the fact that lenders cannot force borrowers to repay their debts unless the debts are secured.[1] In such an economy, durable assets such as land, buildings and machinery play a dual role: they are not only factors of production, but they also serve as collateral for loans. Borrowers' credit limits are affected by the prices of the collateralized assets. And at the same time, these prices are affected by the size of the credit limits. The dynamic interaction between credit limits and asset prices turns out to be a powerful transmission mechanism by which the effects of shocks persist, amplify, and spread out.

The transmission mechanism works as follows. Consider an economy in which land is used to produce output as well as to secure loans, and the total supply of land is fixed. Some firms are credit constrained, and are highly levered in that they have borrowed heavily against the value of their land holdings, which is their major asset. Other firms are not credit constrained. Suppose that in some period t the firms experience a temporary productivity shock which reduces their net worth. Being unable to borrow more, the credit constrained firms are forced to cut back on their investment expenditure, including investment in land. This hurts them in the next period: they earn less revenue, their net worth falls, and, again because of credit constraints, they reduce investment. The knock-on effects continue,

[1] The specific model of debt which we use is a simple version of that in Hart and Moore (1994).

with the result that the temporary shock in period t reduces the constrained firms' demand for land not only in period t but also in periods t+1, t+2, ... To clear the market in each of these periods, the demand for land by the unconstrained firms has to increase, which requires that their opportunity cost, or user cost, of holding land must fall. Given that these firms are unconstrained, their user cost in each period is simply the difference between that period's land price and the expected discounted value of the land price in the following period. This anticipated decline in user costs in periods t, t+1, t+2, ... is reflected by a fall in the land price in period t -- since price equals the expected discounted value of future user costs.

The fall in land price in period t has a significant impact on the behavior of the constrained firms. They suffer a capital loss on their land holdings, which, because of the high leverage, causes their net worth to drop considerably. As a result, the firms have to make yet deeper cuts in their investment in land. There is an intertemporal multiplier process: the shock to the constrained firms' net worth in period t causes them to cut their demand for land in period t and in subsequent periods; to restore market equilibrium, the unconstrained firms' user cost of land is thus anticipated to fall in each of these periods, which leads to a fall in the land price in period t; and this reduces the constrained firms' net worth in period t still further. Persistence and amplification reinforce each other. The process is summarized in Figure 1.

In fact, two kinds of multiplier process are exhibited in Figure 1, and it is useful to distinguish between them. One is a within-period, or static, multiplier. Consider the left-hand, date t column of Figure 1 (that is, ignore any arrows to and from the future). The productivity shock reduces the net worth of the constrained firms, and forces them to cut back their demand for land; the user cost falls to clear the market; the land price drops by the same amount (keeping the future constant), which lowers the value of the firms' existing land holdings, and reduces their net worth still further. But this simple intuition misses the much more powerful intertemporal, or dynamic, multiplier. The future is not constant. As the arrows to the right of the date t column in Figure 1 indicate, the overall drop in the date t land price is the cumulative fall in present and future

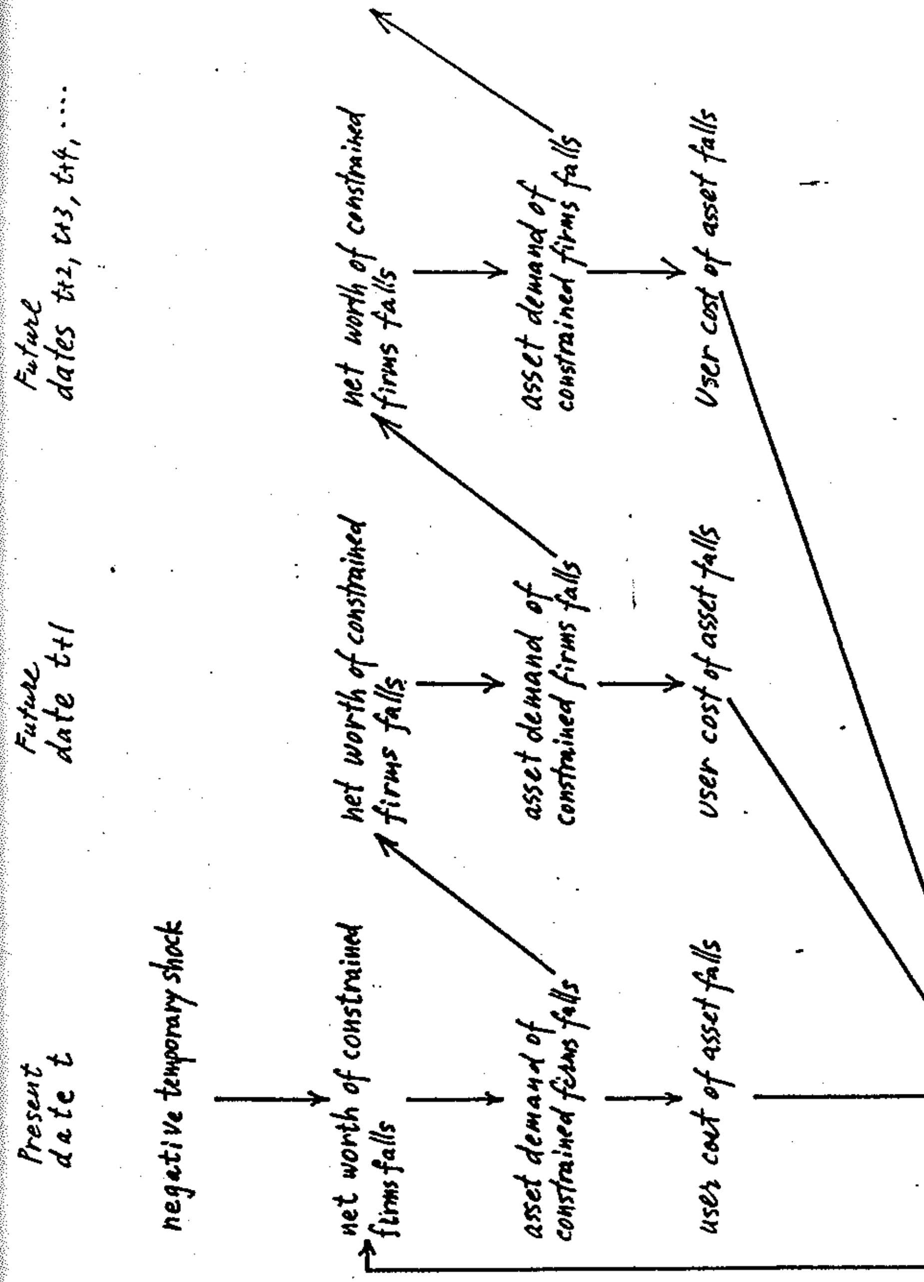

Figure 1

user costs, stemming from the persistent reductions in the constrained firms' net worth and land demand, which are in turn exacerbated by the fall in land price and net worth at date t.

We find that the effect of this dynamic multiplier on land price exceeds that of the static multiplier by a factor equal to the inverse of the net real rate of interest. For our basic model, in percentage terms, the change in land price is of the same order of magnitude as the temporary productivity shock; and the change in land usage exceeds the shock. In the absence of the dynamic multiplier, these changes would be much smaller: the percentage change in price would only be of the order of the shock times the interest rate (i.e. the price would only experience a tiny blip if the length of the period is not long).

A feature of equilibrium is that the marginal productivity of the constrained firms is higher than that of the unconstrained firms -- not surprisingly, given that the constrained firms cannot borrow as much as they want. Consequently, any shift in land usage from the constrained to the unconstrained firms leads to a first-order decline in aggregate output. Aggregate productivity, measured by average output per unit of land, also declines, not because there are variations in the underlying technologies (aside from the initial shock), but rather because the change in land use has a compositional effect.[2]

Our basic model, presented in Section 2, is of a land economy in which all the credit constrained firms are identical. Key to the analysis is the evolution of a representative constrained firm. Its choices in each period are governed by a flow of funds constraint: debt is rolled over; and the returns from previous investments are used not only to pay interest but also to finance downpayments for current investment. There is an unfolding interplay between the past and the future. At any moment, history determines the firm's current revenue, land holdings and debt. But the price of those land holdings -- which affects the firm's net worth and borrowing capacity --

[2]This may shed light on why the aggregate Solow residual fluctuates so much over the business cycle.

is determined by the future. It is the evolution of asset prices, asset holdings, output and borrowing over the business cycle with which we are primarily concerned.

Our full model is in Section 3 of the paper. There are two substantive changes to the basic model of Section 2. First, we allow for investment in a depreciating, reproducible asset which cannot be collateralized. Second, we introduce heterogeneity among the credit constrained firms: in each period some have an investment opportunity so that they borrow up to their credit limits; others are unable to invest and instead use their revenues to pay off some of their debts. As a result, the "aggregate borrowing constraint" of these firms no longer binds: their aggregate borrowing is uncoupled from the value of their aggregate land holdings. We find that in such an economy, a cyclical pattern emerges: recessions lead to booms, and booms lead to recessions. A simple way to understand why the economy cycles is to use the analogy of a predator-prey model. Imagine populations of deer and wolves. If the deer population rises, the wolves that feed on them also multiply. However, as the wolves grow in number, they kill off the deer. Eventually, the deer population falls, which means that fewer wolves can survive. But with fewer wolves, the deer population can in time start to grow again; and so on. That is, away from steady state, the two populations cycle. Now the deer correspond to the land holdings of the credit constrained firms, and the wolves correspond to their debts. On the one hand, a rise in these firms' land holdings means that they have more net worth with which to borrow: the deer feed the wolves. On the other hand, a high level of debt erodes the firms' available funds and curtails their investment in land: the wolves kill off the deer.[3] Our model is actually more complicated than this, because in addition to the deer (the land holdings of the credit constrained firms) and the wolves (the firms' debt), we have a third state variable, the price of land. We find that the land price leads the fluctuations in output.

The role of the uncollateralized asset is examined in Section 4 of the paper. We show that it causes the effects of a shock to persist longer, and

[3]For some interesting economic applications of the predator-prey model, see Das (1993).

it also magnifies the movement of asset prices relative to the movement of quantities. Section 5 reports on numerical simulations of the model.

In Section 6 we extend our basic model to have more sectors, to see to what extent shocks spill over through the (common) land market. Suppose the credit constrained firms in one sector suffer a productivity shock in period t. Then credit constrained firms in other sectors are affected too, even though they may not have experienced any shock of their own. The reason is that the fall in land price in period t reduces their net worth, and they are forced to cut back on investment, both in period t and in the future. What is interesting is the magnitude of this contagion. Given high leverage, it turns out the indirect effects of the fall in land price dwarf the direct effects of the shock, and as a result there is significant co-movement across sectors.

We make some final remarks in Section 7. The rest of the Introduction is taken up with a brief look at the related literature.

Related Literature

The ideas in this paper can be traced at least as far back as Veblen (1904, Chapter V), who clearly describes the positive interactions between asset prices and collateralized borrowing. Moreover, in discussing how the "extension of loans on collateral ... has ... a cumulative character" (page 106), he hints at the dynamic multiplier process we outlined above.[4]

The theoretical literature on financial structure and aggregate economic activity is vast, and it would be unwise to attempt to review it here.[5] We have picked out for discussion four papers which are relevant to

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[4] Much of Veblen's concern is with the mis-pricing of assets, arising from conflicts of interest between producers and financiers.

[5] Gertler (1989) has written an excellent survey, which not only identifies and clarifies the issues, but also provides an account of the historical developments.

our ideas.

Bernanke and Gertler (1989) construct an overlapping generations model in which financial market imperfections cause temporary shocks in net worth to be amplified and persist.[6] All agents earn a spot wage when young, which they then invest for their old age. Some agents -- entrepreneurs -- have access to projects, which require outside finance over and above the inside finance their own labor income provides. This outside finance is supplied by the other young agents. Financial contracts are imperfect because project returns can only be verified at a cost, and so only a limited number of the better projects are funded: agency problems prevent some inherently profitable projects from being undertaken. The projects that do go ahead provide employment for the next generation of young agents. Now it is easy to see why this economy exhibits amplification and persistence. A positive technology shock, for example, not only increases the labor demanded by entrepreneurs who have been funded, but also allows for more projects to be undertaken. Moreover, the accompanying rise in wage improves the financial position of the next generation of entrepreneurs, so more of their projects will be funded too, they will subsequently demand more labor, and so on. In short, a business upswing improves net worth and lowers the cost of agency, which in turn accentuates and perpetuates the upswing; and vice versa for downswings.

Aside from matters of modelling strategy,[7] our model adds quite a kick to the Bernanke and Gertler story. At business cycle frequencies, a major channel for shocks to net worth is through changes in the values of firms' assets and/or liabilities -- particularly when firms are highly levered. Asset prices reflect future market conditions. When the effects of a shock persist (as they do in Bernanke and Gertler), the cumulative impact on asset

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[6] Greenwald and Stiglitz have pursued a similar line of enquiry; see, for example, their 1993 paper.

[7] We think that a model of debt which is based on control over assets, rather than on the cost of verifying project returns, is both more compelling and considerably simpler, especially when extended to more than two periods. (For a multi-period model of debt with costly state verification, see Gertler (1992).)

prices, and hence on net worth at the time of the shock, can be significant. This positive feedback through asset prices, and the associated intertemporal multiplier process, are the key innovations in our paper.[8]

Kashyap, Scharfstein and Weil (1990) present a two-period general equilibrium model in which the scale of aggregate investment in period 1 is governed by the value of collateral -- land -- in period 2. Asymmetric information about firms' prospects prevents first-best financing, and land provides lenders with security. The price of land in period 2 is determined by the level of consumption demand by the firms' shareholders, which is in turn affected by the firms' profitability. Multiple equilibria can exist: optimism about the period 2 land price means that more firms borrow in period 1, which boosts aggregate profit, dividends and hence consumer demand for land in period 2, and so rationalizes the optimism about the land price; and the reverse holds mutatis mutandis. This two-period model shares with our model the feature that there is a positive feedback in both directions between borrowing limits and the price of collateral. However, a two-period model cannot capture the important dynamic multiplier process.[9]

The two-way feedback between borrowing limits and the price of assets connects with the paper by Shleifer and Vishny (1992) on debt capacity. Their argument is that when a firm in financial distress liquidates assets, the natural purchasers are other firms in the same industry. But if one firm is experiencing hard times, it is likely that the other firms in the industry will be too, and so demand for liquidated assets will be lower. The concomitant fall in asset price exacerbates the problem by lowering the debt capacity of firms. Again, the essentially static nature of this argument hides what we consider to be the crucial interplay between persistence and

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[8] A paper on trading in the housing market by Stein (1993) shows that, because of leverage, an increase in the price of housing can increase net worth by more than the required downpayment, leading to an increase in demand for houses. That is, as in our model, asset demand schedules can slope upwards. Although Stein's model is not dynamic, his paper provides an interesting explanation for the observed correlation between price and trading volume in the housing market.

[9] Similar remarks can be made with regard to the two-period multiple equilibrium model of Lamont (1992).

amplification.

Finally, Scheinkman and Weiss (1986) construct a dynamic general equilibrium model in which two types of infinitely-lived agents switch over time between two states, productive and unproductive, following an exogenous stochastic process. Instead of a complete market structure which would insure the risk so as to smooth consumption and aggregate output completely, there is only one market in which a durable, nonproduced asset is traded for the nondurable consumption good; no borrowing is allowed. Individuals buy and sell this asset, adjusting labor hours, output, consumption and saving in order to partially self-insure against the productivity shock. Scheinkman and Weiss show that aggregate output and the asset price fluctuate over time with the evolution of the asset distribution. The reason is that, pending a switch of roles, as the productive agents hold more of the asset, they become less eager to work to produce output and exchange it for the asset, and the unproductive agents become more reluctant to sell the asset to get the consumption good. Notice that in this model the asset is a means of precautionary saving, and only the net positions of agents matter: there are no leverage effects. By contrast, in our model, the asset is not only a means of saving, but also a factor of production. Further, the asset provides security for borrowing; so the price of the asset affects borrowing limits. Because an agent needs the asset in order to produce, he holds a levered position -- which make his net worth very vulnerable to changes in the asset price.

There is some empirical evidence to support the view that investment decisions are not solely determined by the net present value of new projects, but are also affected by an investing firm's balance sheet position and the value of its collateralized assets. See, for example, Black and de Meza (1992); Black, de Meza and Jeffreys (1992); Evans and Jovanovic (1989); Fazzari, Hubbard and Peterson (1988); Gertler and Gilchrist (1994); Holtz-Eakin, Joulfaian and Rosen (1994); Hoshi, Kashyap and Scharfstein (1991); Hubbard and Kashyap (1992); and Whited (1992). Concerning housing investment, the study by Caplin, Freeman and Tracy (1994) looks at panel data on mortgages and shows how, following a fall in interest rates, refinancing is adversely affected if collateral constraints are tightened by depressed real estate prices. At the aggregate level, a number of studies have highlighted the importance of credit constraints in explaining fluctuations in activity; see in particular Bernanke (1983); Eckstein and Sinai (1986); and Friedman (1986).[10]

2. The Basic Model: Amplification and Persistence

Consider a discrete time economy with two goods, a durable asset and a nondurable commodity. It is helpful to think of the durable asset as land, which does not depreciate and has a fixed total supply of $\bar{K}$. The nondurable commodity may be thought of as fruit, which grows on land but cannot be stored. There is a continuum of agents. Some are farmers, some are gatherers, with population sizes 1 and m respectively. Both farmers and gatherers produce and eat fruit. Everyone lives forever and has the same risk neutral preferences:

$$(1) \qquad E_t \left(\sum_{s=0}^{\infty} R^{-s} x_{t+s} \right),$$

where x_{t+s} is consumption of fruit at date t+s, the constant $R > 1$ is one plus the rate of time preference, and E_t denotes expectations formed at date t.

At each date t there is a competitive spot market in which land is exchanged for fruit at a price of q_t. (Throughout, fruit is taken as the numeraire.) The only other market is a one-period credit market in which one unit of fruit at date t is exchanged for a claim to R_t units of fruit at date t+1. The linearity of preferences in (1) implies that the real rate of interest always equals the rate of time preference, i.e. $R_t \equiv R$.

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[10] Land values have experienced huge fluctuations in Japan, where a large part business investment is financed by bank loans secured against real estate. To give some idea of a recent episode: the capital gains from 1986 to 1990, and then the capital losses from 1990 to 1994, were both in excess of annual GDP. Significant swings in the value of real estate, accompanying the fortunes of the general economy, have also recently been experienced in, for example, Scandinavia, southern England, New England, Texas, and California.

Both farmers and gatherers take one period to produce fruit from land, but the farmers differ from the gatherers in their production technologies. We begin with the farmers, since they play the central role in the model. Consider any particular farmer. He or she has a constant returns to scale production function:

$$(2) \qquad y_{t+1} = F(k_t) \equiv (a + c)k_t,$$

where k_t is the land used at date t, and y_{t+1} is the output of fruit at date t+1. Only ak_t of this output is tradeable in the market, however. The rest, ck_t, is bruised and cannot be transported, but can be consumed by the farmer. We introduce nontradeable output in order to avoid the situation where the farmer continually postpones consumption. The ratio a/(a+c) may be thought of as a technological upper bound on his savings rate, which we take to be less than 1/R; that is, we assume

$$(A1) \qquad c > (R-1)a.$$

This inequality is a weak assumption, since R is typically close to 1.

There are two further critical assumptions we make about farming. First, we assume that each farmer's technology is idiosyncratic, in the sense that, once his production has started at date t with land k_t, say, only he has the skill necessary for the land to bear fruit at date t+1. That is, if the farmer were to withdraw his labor between dates t and t+1, there would be no fruit output at date t+1, there would only be the land k_t. Second, we assume that a farmer always has the freedom to withdraw his labor; he cannot precommit to work. In the language of Hart and Moore (1994), the farmer's human capital is inalienable.

The upshot of these two assumptions is that if a farmer has a lot of debt he may find it advantageous to threaten his creditors by withdrawing his labor and repudiating his debt contract. Creditors protect themselves from the threat of repudiation by collateralising the farmer's land. However, because the land yields no fruit without the farmer's labor, the liquidation value (the outside value) of the land is less than what the land would earn under his control (the inside value). Thus, following a repudiation, it is efficient for the farmer to bribe his creditors into letting him keep the land. In effect, he can renegotiate a smaller loan. The division of surplus in this renegotiation process is moot, but Hart and Moore give an argument to suggest that the farmer may be able to negotiate the debt down to the liquidation value of the land.[11] Creditors know of this possibility in advance, and so take care never to allow the size of the debt (gross of interest) to exceed the value of the collateral.[12] Specifically, if at date t the farmer has land k_t, then he can borrow b_t in total, as long as the repayment does not exceed the market value of his land at date t+1:

[11] The case we have in mind is where the liquidation (outside) value is greater than the share of the continuation (inside) value that creditors would get if the liquidation option were not available to them -- albeit that the liquidation value is less than the total continuation value. In this case, the creditors' "outside option" (the option to liquidate) is binding, which pins down the division of surplus in the renegotiation process. For a discussion of the noncooperative foundations of the so-called Outside Option Principle, see Section 3.12 of Osborne and Rubinstein (1990). See the Appendix to Hart and Moore (1994) for specific details of the debt renegotiation game. (Notice that we allow the farmer to repudiate between the time at which the loan is taken out, date t, and the time at which the fruit is harvested, date t+1. In the context of the Hart-Moore analysis, our "periods" are effectively broken into two half periods.)

[12] An alternative, somewhat starker, form of moral hazard would be to assume that the farmers can steal the fruit crop at date t+1 (see Hart and Moore (1989)). In our basic model, this simple diversion assumption leads to the same borrowing constraint: creditors must never allow a farmer's debt obligations to rise above the value of his land, otherwise he will simply abscond, leaving the land behind but taking all the fruit with him. We have chosen not to tell the story this way, because in our full model given in Sections 3 and 4 there are specific trees growing on the land, which are valuable to the farmer but not to outsiders. Were stealing fruit the only moral hazard problem, the farmer would be able to collateralize his trees as well as his land (since, if he absconded, he would have to leave the trees behind). We are interested in investigating the role of an uncollateralized asset (trees), so we want the farmer only to be able to put up his land as security.

$$Rb_t \leq q_{t+1}k_t. \tag{3}$$

Note that there is no aggregate uncertainty in our model (aside from an initial unanticipated shock), and so, given rational expectations, agents have perfect foresight of future land prices.[13]

The farmer can expand his scale of production by investing in more land. Consider a farmer who at the end of date t-1 holds k_{t-1} land, and has incurred a total debt of b_{t-1}. At date t he harvests ak_{t-1} tradeable fruit, which, together with a new loan b_t, is available to cover the cost of buying new land, to repay the accumulated debt Rb_{t-1} (which includes interest), and to meet any additional consumption $x_t - ck_{t-1}$ (over and above the automatic consumption of nontradeable output ck_{t-1}). The farmer's flow of funds constraint is thus

[13] Readers may wonder why farmers cannot find some other way to raise capital, e.g. by issuing equity. Unfortunately, given the specific nature of a farmer's technology, and the fact that he can withdraw his labor, equity holders could not be assured that they would receive a dividend. Debt contracts secured on the farmer's land are the only financial instrument which investors can rely on. The same considerations rule out partnerships between farmers, or larger farming cooperatives.

Longer term debt contracts also offer no additional source of capital, insofar as the farmer can repudiate and renegotiate at any time during the life of a contract. To avoid repudiation and renegotiation, creditors have to ensure that the value of their outstanding loan never exceeds the current liquidation value of the land, i.e. that (3) holds at all times. This means that any credible long-term debt contract can be mimicked by a sequence of short term debt contracts.

It is worth remarking that if land were rented rather than purchased, this would not change production or allocation along the perfect foresight equilibrium path of the economy (although the economy would react differently to unanticipated aggregate shocks). We choose to rule out a rental market for land, because in our full model in Section 3 farmers plant trees on land, and each farmer's trees are specific to him. If land were rented period-by-period, then a farmer would be at the mercy of the landlords who own the land on which his specific trees are growing. Given that, along the equilibrium path, the farmer can buy just as much land as he can rent, he is better off purchasing the land outright, so as to avoid being held up by landlords.

$$q_t(k_t - k_{t-1}) + Rb_{t-1} + x_t - ck_{t-1} = ak_{t-1} + b_t. \tag{4}$$

We turn now to the <u>gatherers</u>. Each gatherer has an identical production function which exhibits decreasing returns to scale: an input of k'_t land at date t yields y'_{t+1} tradeable fruit at date t+1, according to

$$y'_{t+1} = G(k'_t), \quad \text{where } G' \geq 0,\ G'' < 0,\ G'(0) > aR > G'(\tfrac{\bar{K}}{m}). \tag{5}$$

The last inequalities in (5) are included to avoid corner solutions; i.e. to ensure that both farmers and gatherers are producing (in the neighborhood of the steady state equilibrium). Gatherers' production does not require any specific skill; nor do they produce any nontradeable output. As a result, no gatherer is credit constrained. A gatherer's budget constraint at date t is

$$q_t(k'_t - k'_{t-1}) + Rb'_{t-1} + x'_t = G(k'_{t-1}) + b'_t, \tag{6}$$

where x'_t is consumption at date t, Rb'_{t-1} is debt repayment, and b'_t is new debt. In equilibrium, b'_{t-1} and b'_t are actually negative, reflecting the fact that gatherers are creditors to the farmers.

<u>Market equilibrium</u> is defined as a sequence of land prices, and allocations of land, debt and consumption of farmers and gatherers, $\{q_t, k_t, k'_t, b_t, b'_t, x_t, x'_t\}$, such that: each farmer chooses $\{k_t, b_t, x_t\}$ to maximize the expected discounted utility (1) subject to the production function (2), the borrowing constraint (3), and the flow of funds constraint (4); each gatherer chooses $\{k'_t, b'_t, x'_t\}$ to maximize the expected discounted utility (1) subject to the production function (5) and the budget constraint (6); and the markets for land, fruit and debt clear.

To characterize equilibrium, we first examine the farmers' behavior. If the present value of future returns (sum of tradeable and nontradeable output) exceeds the land price,

$$\sum_{s=1}^{\infty} R^{-s}(a + c) > q_t, \tag{7}$$

then a farmer will always prefer to borrow up the maximum so as to expand land k_t, and consume no more than the automatic consumption of nontradeable fruit ck_{t-1}.[14] Later we show that condition (A1) implies that (7) holds in the neighborhood of the steady state equilibrium, and so the farmer's optimal choice of $\{k_t, b_t, x_t\}$ satisfy $x_t = ck_{t-1}$ in (4) and the borrowing constraint (3) is binding. That is,

$$k_t = \frac{1}{q_t - \frac{1}{R}q_{t+1}} \left((a + q_t)k_{t-1} - Rb_{t-1} \right) \tag{8}$$

$$b_t = \frac{1}{R}q_{t+1}k_t. \tag{9}$$

The term $\left((a + q_t)k_{t-1} - Rb_{t-1}\right)$ is the farmer's net worth at the beginning of date t, were his assets to be liquidated; i.e. the value of his tradeable output and land held from the previous period, net of debt repayment and interest. In effect, (8) says that the farmer uses his net worth to finance the difference between the price of land, q_t, and the amount he can borrow against each unit of land, $\frac{1}{R}q_{t+1}$. This difference, $q_t - \frac{1}{R}q_{t+1}$, can be

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[14] Strictly speaking, future returns can be expressed simply as the sum of tradeable and nontradeable output if and only if the borrowing constraint is <u>not</u> binding. But the borrowing constraint is <u>not</u> binding if and only if

$$\sum_{s=1}^{\infty} R^{-s}(a + c) \leq q_t. \tag{†}$$

That is, the borrowing constraint is binding if and only if (†) is not satisfied, i.e. (7) holds.

thought of as the downpayment required to purchase a unit of land.

Since equations (8) and (9) are linear and hold for every farmer, they also hold for the aggregate land holding and borrowing, K_t and B_t say, of the farming sector:

$$(10) \qquad K_t = \frac{1}{q_t - \frac{1}{R}q_{t+1}} \left((a + q_t)K_{t-1} - RB_{t-1} \right)$$

$$(11) \qquad B_t = \frac{1}{R}q_{t+1}K_t.$$

Notice from (10) that if, for example, present and future land prices, q_t and q_{t+1}, were to rise in equal proportion, then the farmers' demand for land at date t would also rise -- provided that leverage is sufficient that debt repayments RB_{t-1} exceed current output aK_{t-1} (which will be true in equilibrium). The usual notion that a higher land price q_t reduces the farmers' demand is more than offset by the facts that (i) they can borrow more when q_{t+1} is higher; and (ii) their net worth increases as q_t rises. Even though the required downpayment, $q_t - \frac{1}{R}q_{t+1}$, per unit of land rises proportionately with q_t and q_{t+1}, the farmers' net worth is increasing more than proportionately with q_t due to the leverage effect of the outstanding debt.

Next we examine the gatherers' behavior. A gatherer is not credit constrained, and so his or her demand for land is determined at the point where the present value of the marginal product of land is equal to the opportunity cost, or user cost, of holding land, u_t say:

$$(12) \qquad \frac{1}{R}G'(k'_t) = u_t \equiv q_t - \frac{1}{R}q_{t+1}.$$ [15]

[15]The user cost u_t can be broken into two components: the interest component, $(R-1)q_t/R$, minus the capital gains component, $(q_{t+1}-q_t)/R$.

Notice the dual role played by u_t in the model: not only is u_t the gatherers' opportunity cost of holding a unit of land (the right hand side of (12)), but also u_t happens to be the required downpayment per unit of land held by the farmers (the denominator on the right hand side of (8) and (10)).[16]

Finally, we consider market clearing. Since all the gatherers have identical production functions, their aggregate demand for land, K'_t say, is equal to k'_t times their population m. The sum of the aggregate demands for land by the farmers and gatherers is equal to the total supply; i.e. $K_t + K'_t = \bar{K}$. Thus, from (12), we obtain the land market equilibrium condition

$$(13) \qquad q_t - \frac{1}{R}q_{t+1} = u(K_t), \qquad \text{where } u(K) \equiv \frac{1}{R}G'\left(\frac{1}{m}(\bar{K} - K)\right).$$

The function $u(\cdot)$ is increasing: if the farmers' demand for land, K_t, goes up, then in order for the land market to clear the gatherers' demand has to be choked off by a rise in the user cost, u_t. Since the interest rate equals the rate of time preference R-1, the gatherers are indifferent about any path of consumption and debt (or credit); and so, given (13), the markets for fruit and credit are in equilibrium by Walras' law.

We restrict attention to perfect foresight equilibria in which, without

[16]We say "happens to be" because a different borrowing constraint from (3) would yield different expressions for k_t and K_t in (8) and (10). For example, we might suppose that, out of equilibrium, if a farmer were to repudiate his debt contract and the renegotiation of a new contract with his creditors were to break down, then there would be a (proportional) transactions cost τ associated with disposing of his land. That is, the liquidation value would be multiplied by $(1-\tau)$. The borrowing constraint (3) would become $Rb_t \leq (1-\tau)q_{t+1}k_t$, and the denominator on the right hand side of (8) and (10) would read $q_t - \frac{1-\tau}{R}q_{t+1}$. Although the analysis of the model would then be slightly different, its behavior would be similar.

unanticipated shocks, the expectations of future variables realize themselves. For a given level of farmers' land holding and debt at the previous date, K_{t-1} and B_{t-1}, an equilibrium from date t onwards is characterized by the path of land price, farmers' land holding and debt, $\{(q_{t+s}, K_{t+s}, B_{t+s}) | s \geq 0\}$, satisfying equations (10), (11) and (13) at dates t, t+1, t+2, ... We also rule out exploding bubbles in the land price:

$$\text{(A2)} \qquad \lim_{s\to\infty} E_t\,(R^{-s}q_{t+s}) = 0.$$

Given (A2), it turns out that there is a locally unique perfect foresight equilibrium path starting from initial values K_{t-1} and B_{t-1} in the neighborhood of the steady state.[17]

Before we turn to dynamics, it is useful to look at the steady state equilibrium. From equations (10), (11) and (13), it is easily shown that there is a unique steady state, (q^*, K^*, B^*), with associated steady state user cost u^*, where

$$\text{(14a)} \qquad \frac{R-1}{R}q^* = u^* = a$$

$$\text{(14b)} \qquad \frac{1}{R}G'\left(\frac{1}{m}(\bar{K} - K^*)\right) = u^*$$

$$\text{(14c)} \qquad B^* = \frac{a}{R-1}K^*$$

In steady state, the farmers' tradeable output, aK^*, is just enough to cover the interest on their debt, $(R-1)B^*$. Equivalently, the required downpayment per unit of land, $q^* - \frac{1}{R}q^*$, equals the farmers' productivity of tradeable output, a. As a result, farms neither expand nor shrink. Notice that, given

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[17] We discuss global properties in footnote 22 below.

assumption (A1),

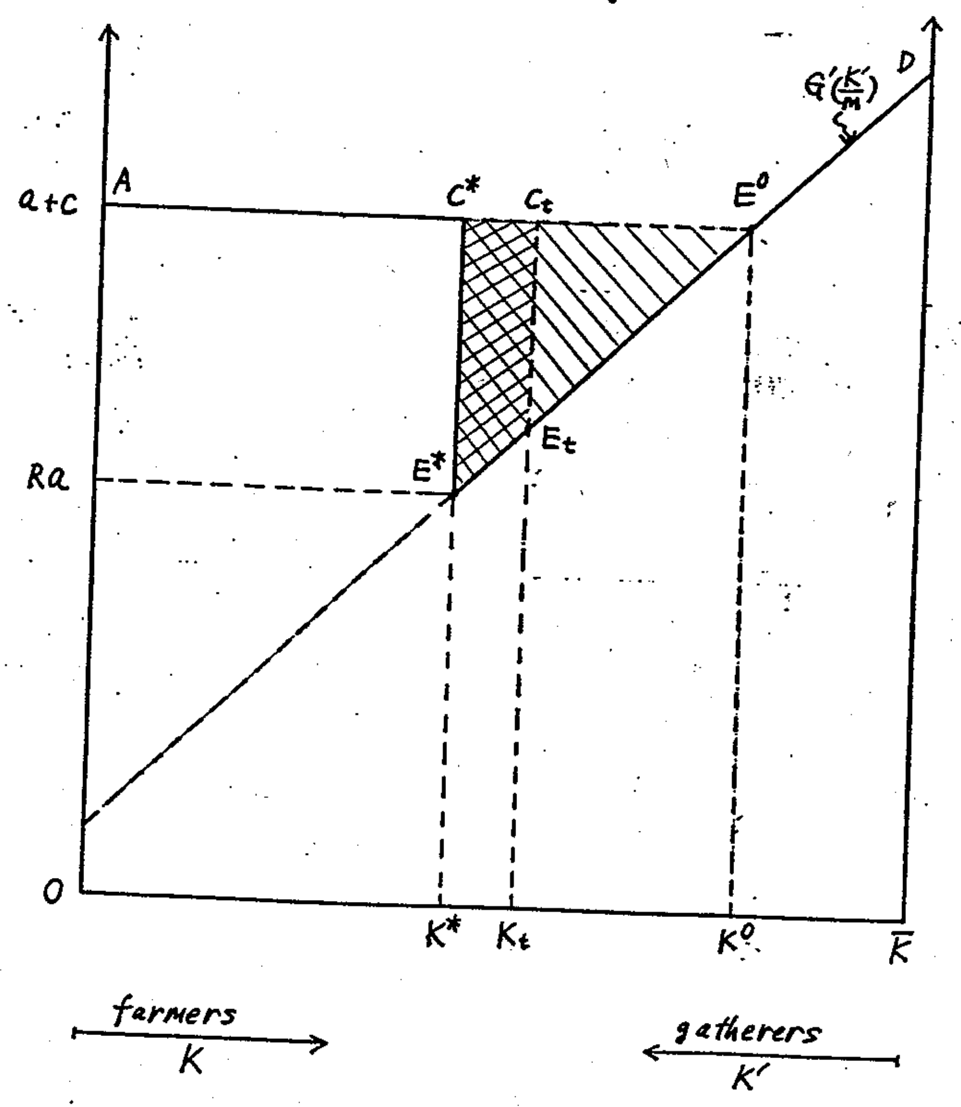
$$\text{(15)} \qquad \frac{a}{R-1} < q^* < \frac{a+c}{R-1}.$$

The left hand inequality in (15) says that, in terms of tradeable output alone, farming has a negative present value: per unit of land, the price is greater than the present value of tradeable output. The reason that farmers nevertheless choose to produce is that, once nontradeable output is factored in, farming is profitable: per unit of land, the price is less than the present value of tradeable and nontradeable output (the right hand inequality in (15)). This confirms that, in the neighborhood of the steady state, inequality (7) also holds.

Figure 2 provides a useful summary of the economy. On the horizontal axis, farmers' demand for land is measured from the left, gatherers' demand from the right, and the sum of the two equals total supply $\bar{K}$. On the vertical axes are the marginal products of land. The farmers' marginal product of land equals a+c, indicated by the line AC^*E^o. The gatherers' marginal product is shown by the line DE^oE^*; it falls with their land usage. If there were no debt enforcement problem so that there were no credit constraint, then the first-best allocation would be at the point $E^o = (K^o, a+c)$, at which the marginal products of the farmers and the gatherers would be equalized. The land price would be $q^o = (a+c)/(R-1)$, the discounted gross return from farming. In our credit-constrained economy, the steady state equilibrium is at the point $E^* = (K^*, Ra)$, where the marginal product of the farmers, a+c, exceeds the marginal product of the gatherers, $G'\left(\frac{1}{m}(\bar{K}-K^*)\right) = Ra$. (Assumption (A1) tells us that $a+c > Ra$.) That is, relative to the first-best, in the credit-constrained equilibrium too little land is used by the farmers.

The area under the solid line, AC^*E^*D, is the steady state output, Y^* say, of fruit per period. The triangular shaded area $C^*E^*E^o$ is the output loss relative to the first-best. It is important to observe that, in the neighborhood of the steady state, aggregate output, Y_{t+1} say, at some date t+1 is an increasing function of the farmers' land holding, K_t, at date t.

Figure 2 Steady State



In fact, around K^*, a rise in K_t causes a first-order increase in Y_{t+1}: the area under the solid line in Figure 2 changes by the trapezoid $E^*C^*C_tE_t$.[18]

To understand the dynamics of the economy, we find it helpful to consider the response to an unexpected impulse. Suppose at date t-1 the economy is in steady state: $K_{t-1} = K^*$ and $B_{t-1} = B^*$. There is then an unexpected shock to productivity: both the farmers' and the gatherers' fruit harvests at the start of date t are $1 + \Delta$ times higher than expected. For exposition, we take Δ to be positive. However, the shock is known to be temporary. The farmers' and gatherers' production technologies between dates t and t+1 (and thereafter) return to (2) and (5) respectively.[19]

Combining the market clearing condition (13) with the farmers' demand for land (10) and their borrowing constraint (11) at dates t, t+1, t+2, ... , we obtain

$$u(K_t)\, K_t = (a + \Delta a + q_t - q^*)K^* \qquad \text{(date t)} \tag{16a}$$

$$u(K_{t+s})\, K_{t+s} = aK_{t+s-1} \quad \text{for } s \geq 1 \qquad \text{(dates t+1, t+2, ...)} \tag{16b}$$

Equations (16) say that at each date the farmers can hold land up to the

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[18] This would not be true in the first-best near K^0, where the change in area would only be a triangle: a second-order change in output. (In fact, we shall argue at the end of this section that in the first-best K_t does not respond at all to an unanticipated, temporary output shock at date t, since land demands are entirely determined by current and future prices, which do not change.)

[19] Our analysis of a farmer's borrowing constraint (3) presupposed a deterministic environment. To allow for the possibility of an unexpected shock, we assume that a farmer's labor supply decision is made between periods, before the shock is realized. That is, it is too late for a farmer to repudiate his debt contract after the shock, because by then he has input his labor. Of course, we are relying here on the fact that the shock is a genuine surprise, and that the debt contract is not contingent; for further discussion, see Section 7.

point K where the required downpayment, $u(K)K$, is covered by their net worth. Notice that in (16b), at each date $t+s$ ($s \geq 1$), the farmers' net worth is just their current output of tradeable fruit, aK_{t+s-1}: from the borrowing constraint at date $t+s-1$, the value of the farmers' land at date $t+s$ is exactly offset by the amount of debt outstanding. In (16a), however, we see that the farmers' net worth at date t -- just after the shock hits -- is more than simply their current output, $(1 + \Delta)aK^*$, because q_t jumps in response to the shock and they enjoy unexpected capital gains, $(q_t - q^*)K^*$, on their land holdings (the value of land held from date t-1 is now q_tK^*, while the debt repayment is $RB^* = q^*K^*$).

To find closed-form expressions for the new equilibrium path, we take Δ to be small and linearize around the steady state. Let $\hat{X}_t$ denote the proportional change, $(X_t - X^*)/X^*$, in a variable X_t relative to its steady state value X^*. Then, using the fact that $(R-1)q^*/R = u^* = a$, equations (16) become

$$\text{(17a)} \qquad \left(1 + \frac{1}{\eta}\right) \hat{K}_t = \Delta + \frac{R}{R-1} \hat{q}_t \qquad \text{(date t)}$$

$$\text{(17b)} \qquad \left(1 + \frac{1}{\eta}\right) \hat{K}_{t+s} = \hat{K}_{t+s-1} \quad \text{for } s \geq 1 \qquad \text{(dates t+1, t+2, ...)}$$

-- where $\eta > 0$ denotes the elasticity of the residual supply of land to the farmers with respect to the user cost at steady state.[20]

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[20] That is,

$$\frac{1}{\eta} = \left.\frac{d\log u(K)}{d\log K}\right|_{K=K^*} = - \left.\frac{d\log G'(k')}{d\log k'}\right|_{k'=\frac{1}{m}(\bar{K}-K^*)} \times \frac{K^*}{(\bar{K} - K^*)}$$

-- which is the elasticity of the gatherers' marginal product of land times the ratio of the farmers' to the gatherers' land holdings in steady state. η is positive, given our assumption that $G'' < 0$.

The right hand side of (17a) divides the change in the farmers' net worth at date t into two components: the direct effect of the productivity shock, Δ; and the indirect effect of the capital gain arising from the unexpected rise in price, $\hat{q}_t$. Crucially, the impact of $\hat{q}_t$ is scaled up by the factor $R/(R-1)$. This is because of leverage: the farmers' steady state net worth equals aK^*; and so, ceteris paribus, a one per cent rise in q_t increases their net worth by $(q^*K^*/aK^*) = R/(R-1)$ per cent.

The factor $\left(1 + \frac{1}{\eta}\right)$ on the left hand sides of (17) reflects the fact that as the farmers' land demand rises, the user cost must rise to clear the market; and this in turn partially chokes off the increase in the farmers' demand. (The less elastic the supply (the lower η), the more the farmers' demand is choked off.) The key point to note from (17b) is that, except for the limit case of a perfectly inelastic supply ($\eta = 0$), the effect of a shock to K_t persists into the future:

$$\text{(18)} \qquad \hat{K}_{t+s} = \left(\frac{\eta}{1+\eta}\right)^s \hat{K}_t \quad \text{for } s \geq 1.$$

Persistence occurs because the farmers' ability to invest at each date t+s is determined by how much downpayment they can afford from their net worth at that date, which in turn is historically determined by their level of production at the previous date t+s-1.

It remains to find out the size of the initial change in the farmers' land holdings, $\hat{K}_t$, which, from (17a), is jointly determined with the change in land price, $\hat{q}_t$. Solving forward the land market equilibrium condition (13), using (A2), we know that land price is the discounted sum of future user costs:

$$\text{(19)} \qquad q_t = \sum_{s=0}^{\infty} R^{-s} u(K_{t+s}).$$

(19) reflects the forward-looking nature of the equilibrium: agents foresee that K_{t+s} will continue to be higher than K^* in the future, and therefore

rationally expect that the market-clearing user costs $u(K_{t+s})$ will be higher than u^*. These expectations are priced into q_t. To see how, linearize (19) around the steady state, and then substitute from (18):

$$\text{(20)} \qquad \hat{q}_t = \frac{1}{\eta}\frac{R-1}{R}\sum_{s=0}^{\infty} R^{-s}\hat{K}_{t+s} = \frac{1}{\eta}\frac{R-1}{R}\frac{1}{1-\frac{1}{R}\frac{\eta}{1+\eta}}\hat{K}_t.$$

Here, for the land price q_t to move significantly, both the history-dependent behavior of the farmers and the forward-looking nature of the equilibrium are important. If the farmers' land holdings were not history-dependent, and thus the increase did not persist beyond $\hat{K}_t$, there would only be a single blip in the user cost at date t. The impact on q_t would be scaled down by the interest rate factor $(R-1)/R$; that is, there would only be a small contemporaneous effect of $\hat{K}_t$ on land price. Equally, if the equilibrium were not forward-looking, persistent increases in the farmers' future land holdings $\hat{K}_{t+s}$ would not be priced (via future user costs u_{t+s}) into q_t. The multiplier $\left(1 - \frac{1}{R}\frac{\eta}{1+\eta}\right)^{-1}$ in (20) reflects this interaction between history and future. Although the multiplier may not look large (e.g., η may be small), it has a dramatic effect on the sizes of $\hat{q}_t$ and $\hat{K}_t$, as we now see.

To find $\hat{q}_t$ and $\hat{K}_t$ in terms of the size of the shock Δ, solve (17a) and (20):

$$\text{(21)} \qquad \hat{q}_t = \frac{1}{\eta}\Delta$$

$$\text{(22)} \qquad \hat{K}_t = \frac{1}{1+\frac{1}{\eta}}\left(1 + \frac{R}{R-1}\frac{1}{\eta}\right)\Delta.$$

(21) tells us that, in percentage terms, the effect on the land price at date t is of the same order of magnitude as the <u>temporary</u> productivity shock! As a result, the effect of the shock on the farmers' land holdings at date t is large: the multiplier in (22) exceeds unity, and can do so by a sizeable

margin, thanks to the factor $R/(R-1)$. In terms of (17a), the indirect effect of $\hat{q}_t$, scaled up by the leverage factor $R/(R-1)$, is easily enough to ensure that the overall effect on $\hat{K}_t$ is more than one-for-one. (On its own, the direct effect of the productivity shock Δ has a less than one-for-one effect on $\hat{K}_t$ because a rise in user cost u_t chokes it off.)[21,22]

Recall the distinction we drew in the Introduction between the static and dynamic multipliers. Imagine, hypothetically, that there were no dynamic multiplier. That is, suppose q_{t+1} were artificially pegged at the steady level q^*. (17a) would remain unchanged. However the right hand side of (20) would only contain the first term of the summation -- the term relating to the change in user cost at date t -- so that the multiplier $\left(1 - \frac{1}{R}\frac{\eta}{1+\eta}\right)^{-1}$ would disappear. Combining the modified equation, $\hat{q}_t = \frac{1}{\eta}\frac{R-1}{R}\hat{K}_t$, with (17a), we solve for $\hat{q}_t$ and $\hat{K}_t$:

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[21]For inelastic supply (η less than 1), the indirect effect is particularly marked. We know from (21) that a one percent productivity shock leads to a more than one percent increase in land price. However, the effects are shorter-lived: from (17b) we see that the decay factor is $\eta/(1+\eta) < 1/2$. Conversely, for elastic supply the indirect effect is less marked, and the impact on land price is less than proportional; however, there is more persistence. In the limit, as η approaches ∞, there is no indirect effect: a one percent productivity shock leads to a one percent change in the farmers' land holdings, and there is no change in land price; but there is complete persistence.

[22]Because of the large multiplier effects, the nonlinear equilibrium system (16,19) can have multiple dynamic equilibria, even though the linearized system (17,20) has a unique equilibrium. Solving (16b) as $K_{t+s} = \varphi(K_{t+s-1})$ or $K_{t+s} = \varphi^s(K_t)$, we can combine the equilibrium conditions (16a,19) as:

$$(\dagger\dagger) \qquad u(K_t)K_t - (a + \Delta a)K^* - \sum_{s=0}^{\infty} R^{-s}\Big(u(\varphi^s(K_t)) - a\Big)K^* = 0.$$

($\dagger\dagger$) can have a solution K_t outside a neighbourhood of K^*. (This can be true <u>even when there is no shock</u> ($\Delta = 0$).) In particular, if $Ru(0) < a$, then there is another solution K_t which is considerably less than K^*. Intuitively, if the farmers' future land holdings are expected to be small, then currently the land price will be low, the farmers will have little net worth, and they will be unable to borrow much to buy land -- which in turn justifies the expectation that their future land holdings will be small.

$$(21') \qquad \hat{q}_t \Big|_{q_{t+1}=q^*} = \frac{R-1}{R}\frac{1}{\eta}\Delta$$

$$(22') \qquad \hat{K}_t \Big|_{q_{t+1}=q^*} = \Delta.$$

These are the changes in the land price and the farmers' land holdings which can be traced to the static mutiplier alone. Subtracting (21') from (21), we find that the additional movement in land price attributable to the dynamic multiplier is $\frac{1}{R-1}$ times the movement due to the static multiplier. And a comparison of (22) with (22') shows that the dynamic multiplier has a similarly large proportional effect on the farmers' land holdings.[23]

As we saw in Figure 2, aggregate fruit output -- the combined harvest of the farmers and gatherers -- moves together with the farmers' land holdings, since their marginal product is higher than the gatherers'. It is straightforward to show that at each date t+s the proportional change in aggregate output, $\hat{Y}_{t+s}$, is given by

$$(23) \qquad \hat{Y}_{t+s} = \frac{a+c-Ra}{a+c}\,\frac{(a+c)K^*}{Y^*}\,\hat{K}_{t+s-1} \qquad \text{for } s \geq 1.$$

The term $\frac{a+c-Ra}{a+c}$ reflects the difference between the farmers' productivity (equal to a+c) and the gatherers' productivity (equal to Ra near the steady state). The ratio $\frac{(a+c)K^*}{Y^*}$ is the share of the farmers' output. If aggregate productivity were measured by $Y_{t+s}/\bar{K}$, it would be persistently above its steady state level, even though there are no positive productivity shocks after date t. The explanation lies in a composition effect: there is a persistent change in land usage between farmers and gatherers, which is reflected in increased aggregate output.

One interesting issue is how the economy would respond to other kinds of shock. In particular, suppose instead of a temporary productivity shock at date 1, the economy experiences an unanticipated, one-time reduction in the value of debt obligations. This debt reduction has the same qualitative effect as the temporary productivity increase (except that there is no increase in output at the initial date). Quantitatively, however, since the outstanding debt of the farmers is $\frac{R}{R-1}$ times their output of tradeable fruit (in the steady state, $\frac{RB^*}{aK^*} = \frac{R}{R-1}$), a reduction of only $\frac{R-1}{R}$% in the value of their debt obligations is enough to generate the same effects as a 1% temporary productivity shock.[24]

To close this section, let us ask what would happen in the first-best economy, where there are no credit constraints. Consider the effect of the same unanticipated, temporary productivity shock Δ at date t. Aggregate output Y_t would rise by the factor Δ. But there would be no effect on the land price q_t or the land usage K_t; they would stay at q^o and K^o. Nor would there be any change to future prices and production. The point is that in the first-best economy all agents are unconstrained in the credit market, and prices and production are unaffected by changes to net worth. This is in marked contrast to what we have seen happens in the credit-constrained economy, where q_t and K_t (and hence Y_{t+1}) increase significantly, and these increases persist into the future.

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[23] A less artificial way to get $q_{t+1} = q^*$ would be to have a second, negative, productivity shock, $-\Delta$, at date t+1 (anticipated at date t). That is to say, the static multiplier has the same effect as two equal but opposite shocks hitting the economy in succession.

[24] Although our model does not have money, so we cannot analyse monetary policy per se, one possible monetary transmission mechanism would be through the redistribution of wealth between debtors and creditors, as emphasized by Fisher (1934) and Tobin (1980). If debt contracts were uncontingent and nominal, and if an unexpected increase in the money supply increased the nominal price level, then it would reduce the real burden of outstanding debt.

3. The Full Model: Investment and Cycles

The basic model of Section 2 has a number of shortcomings. The only "investment" is in land itself; and although land changes hands between farmers and gatherers, aggregate investment is automatically zero because the total land supply is fixed. Also, the impulse response of the economy to a shock is arguably too dramatic and short lived (especially when the residual supply of land to the farmers is inelastic). This is because the leverage effect is so strong: in steady state the farmers' debt-asset ratio is $1/R$, which is unreasonably high if the length of the period is not long (i.e. R is close to 1). Finally, the simplicity of the model hides certain important dynamics.

In this section we extend the basic model to overcome these shortcomings. There are two substantive changes. First, we introduce reproducible capital, trees, into the farmers' production function. A farmer plants fruit in his land to grow trees, which in turn yield fruit at subsequent periods. Land does not depreciate, but the trees do. As the farmer must replenish his stock of trees by planting more fruit, the planted fruit can be thought of as investment. We will see that aggregate investment is always positive, and fluctuates together with aggregate output and land price. We will also show that, because of the second input in the farming technology, the impulse response of the economy to a shock is less dramatic but lasts longer.

The second substantive change to the model of Section 2 is that we assume that in each period only a fraction of the farmers have an investment opportunity. The other farmers are unable to invest, and instead use their revenues partially to pay off their debts. Ex post, then, farmers are heterogeneous. The probabilistic investment assumption simply captures the idea that, at the level of the individual enterprise, investment in fixed assets is typically occasional and lumpy.[25] Since it is no longer the case that all farmers are borrowing up to their credit limits, in aggregate the value of the farmers' debt repayments is strictly smaller than the value of their collateralized asset, land. We will see that this uncoupling of the farmers' aggregate borrowing from their aggregate land holdings allows for rich dynamic interactions between q_t, K_t and B_t, and can lead to cycles.

To understand the specifics of the model, consider a particular farmer. We say that his land is <u>cultivated</u> if he has trees growing on it. If he works on k_{t-1} units of cultivated land at date t-1, he will produce ak_{t-1} tradeable fruit and ck_{t-1} nontradeable fruit at date t -- just as in Section 2. A fraction $1-\lambda$ of the trees are assumed to die by date t, and so this part of the land is no longer cultivated. This does not mean that the land cannot be used; it may be used by gatherers, or it may be cultivated again, possibly by another farmer.

In order to increase his holding of cultivated land at date t from λk_{t-1} to k_t, he must plant $\phi(k_t - \lambda k_{t-1})$ fruit, as well as acquire $k_t - k_{t-1}$ more land.[26] However, we assume that a new investment opportunity to plant fruit only arises with probability π. With probability $1-\pi$, the farmer is unable to invest, so the scale of his operations is limited to λk_{t-1} and (in equilibrium) he sells off the $(1-\lambda)k_{t-1}$ uncultivated land. We assume that the arrival of investment opportunities is independent both across farmers and through time (hence, because there is a continuum of farmers, there is no aggregate uncertainty).[27]

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[25] For empirical evidence on this, see, for example, Doms and Dunne (1993). Investment by individual firms may be lumpy because of fixed costs -- an idea which clearly warrants a full analysis. However, in the interests of keeping our aggregate model simple, we rely on the assumption of a probabilistic investment opportunity.

[26] That is, the farmer has a one-period Leontief production function. There are two inputs: land and trees, in 1:1 fixed proportion. And there are four outputs: land, trees, tradeable fruit and nontradeable fruit, in fixed proportions $1:\lambda:a:c$. In addition, the farmer has an instantaneous technology for growing trees: ϕ fruit make 1 tree.

It may be that $\lambda k_{t-1} < k_t < k_{t-1}$, in which case the farmer will sell off $k_{t-1} - k_t$ land. Here, the farm is shrinking, because investment does not cover depreciation.

[27] An alternative, possibly more natural, specification of the depreciation process is that the <u>entire stock</u> of trees of an individual farmer dies with probability $1-\lambda$, and survives with probability λ. (For example, there may be

We make three assumptions about the parameters. We assume that the tradeable output is at least enough to replant the depreciated trees:

$$\text{(A3)} \qquad a > (1-\lambda)\phi.$$

We assume that the arrival rate of an investment opportunity is not too small:

$$\text{(A4)} \qquad \pi > 1 - \frac{1}{R}.$$

Finally, we strengthen (A1) to:

$$\text{(A1}'\text{)} \qquad c > \frac{1 - \lambda + \lambda\pi}{\lambda\pi + (1-\lambda)(1-R+\pi R)} (R-1)(a + \lambda\phi).$$

(The denominator on the right hand side of (A1′) is positive, from (A4).) Notice that (A4) and (A1′) are both weak assumptions, given that R is typically close to 1.

We suppose that each farmer grows his own specific trees, and only he has the skill necessary for them to bear fruit (the other farmers do not know how to prune them). This means that, having sunk the cost (in terms of fruit) of growing trees, there is wedge between the inside value to a farmer

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a storm, or a disease.) These shocks are independent across farmers and through time, and are also independent of the arrival of investment opportunities. (In particular, a farmer who loses his stock of trees <u>and</u> cannot invest sells off his land to become a creditor: we see below why the credit constraint (3) is still satisfied, so that the farmer's gross debt repayment does not exceed the value of his land holding.) In aggregate, this alternative specification leads to the same equilibrium paths as the model in the text. We make the assumption that all farmers' trees depreciate nonstochastically at the rate $1-\lambda$ in order to simplify the exposition.

of his cultivated land and the outside value of the land to everyone else. Also, we continue to assume that a farmer's specific human capital is inalienable: he cannot precommit to tend his trees. And so, from the argument given earlier, we deduce that creditors will be unwilling to lend beyond the limit in (3). That is, only land can serve as collateral. Because trees cannot be collateralized, when a farmer invests the ratio of debt to assets will be lower than if, as in the basic model, he did not have to plant trees.[28]

The rest of the model is exactly the same as in Section 2. To sum up, we have made two changes to the basic model. First, the farmer's flow of fund constraint (4) now includes the investment in trees, $\phi(k_t - \lambda k_{t-1})$:

$$\text{(24)} \qquad q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + Rb_{t-1} + x_t - ck_{t-1} = ak_{t-1} + b_t.$$

Second, at each date t, with probability $1-\pi$, a farmer may now face the additional technological constraint

$$\text{(25)} \qquad k_t \le \lambda k_t.$$

(It is worth observing that, at the risk of laboring the exposition, we could have made these changes one at a time. That is, we could have introduced trees into the model without introducing heterogeneity ($\phi > 0$, $\pi = 1$), or vice versa ($\pi < 1$, $\phi = 0$). Our main focus in this section will be on the dynamic implications of heterogeneity: $\pi < 1$. But without trees as a depreciating second input, (25) would be less easy to rationalize: it makes the story more compelling to have $\phi > 0$ given that we are assuming $\pi < 1$. In fact, including a noncollateralized asset has a number of other important consequences for the dynamics of the model; we postpone discussing these

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[28] Note that the <u>aggregate</u> ratio of the farmers' debt to their assets is even lower, given that not every farmer is investing in each period.

until Section 4, where we examine the special case $\phi > 0$, $\pi = 1$.)

To characterize equilibrium, we need to examine the farmers' behavior. Start with a farmer who can invest at date t. As before, he will invest up to the maximum (consuming no more than the nontradeable output $x_t = ck_{t-1}$), if the present value of future returns on land (the sum of the tradeable and nontradeable output, together with the proceeds from land sales) exceeds the unit cost of investment:

$$(26) \qquad \sum_{s=1}^{\infty} R^{-s} \lambda^{s-1} \left[a + c + (1-\lambda)q_{t+s}\right] > q_t + \phi.$$

(26) is the analogous condition to (7). It is straightforward to show that assumption (A1') implies that (26) is satisfied in the neighbourhood of the steady state equilibrium. The credit constraint is thus binding -- (9) holds -- and, from (24),

$$(27) \qquad k_t = \frac{1}{\phi + q_t - \frac{1}{R}q_{t+1}} \left[(a + q_t + \lambda\phi)k_{t-1} - Rb_{t-1} \right].$$

The term in the large brackets is the farmer's net worth. The investing farmer uses his net worth to finance the difference between the unit cost of investment, $\phi + q_t$, and the amount he can borrow against a unit of land, q_{t+1}/R.

Next consider a farmer who cannot invest at date t. Given that condition (26) holds, the farmer chooses to keep as much land as he can, subject to the technological constraint (25); that is,

$$(28) \qquad k_t = \lambda k_{t-1}.$$

He also uses his tradeable output, ak_{t-1}, together with his receipts from land sales, $q_t(1-\lambda)k_{t-1}$, to pay off his debt rather than consume more than the bruised fruit ck_{t-1}. Thus his new level of indebtedness is given by

$$(29) \qquad b_t = Rb_{t-1} - ak_{t-1} - q_t(1-\lambda)k_{t-1}.$$

Of course we need to show that the farmer <u>can</u> borrow this much. Using assumption (A4), it is straightforward to confirm that, in the neighbourhood of the steady state equilibrium, the expressions for k_t and b_t in (28) and (29) always strictly satisfy the borrowing constraint (3).[29] In fact, if by chance an individual farmer has a long history of no opportunity to invest, he may eventually become a net creditor (i.e. his b_t may become negative) -- while his land holding is always positive and declining geometrically at the rate λ.

Expressions (9), (27), (28) and (29) have the great virtue that they are linear in k_{t-1} and b_{t-1}. Hence we can aggregate across farmers and appeal to the law of large numbers to derive the farmers' aggregate land holdings and borrowing, K_t and B_t, without having to keep track of the distribution of the individual farmers' k_t's and b_t's. Since the population of farmers is unity, with a fraction π investing and a fraction $1-\pi$ not investing, we have

$$(30) \qquad K_t = (1-\pi)\lambda K_{t-1} + \frac{\pi}{\phi + q_t - \frac{1}{R}q_{t+1}} \left[(a + q_t + \lambda\phi)K_{t-1} - RB_{t-1} \right].$$

Also, since no farmer consumes more than his nontradeable output, we deduce from the flow of funds constraint (24) that

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[29] This is equivalent to saying that the farmer's land holding after he has been forced to shrink it back (the right hand side of (28)) is strictly less than what it would have been had he been able to invest (the right hand side of (27)).

$$B_t = RB_{t-1} + q_t(K_t - K_{t-1}) + \phi(K_t - \lambda K_{t-1}) - aK_{t-1}. \tag{31}$$

Notice that (30) and (31) generalize (10) and (11) to the case where $\phi > 0$ and $\pi < 1$.

The land market clearing condition for the user cost $u_t = q_t - \frac{1}{R}q_{t+1}$, equation (13), is unchanged. Thus, for predetermined levels of the farmers' land holdings and debt at the previous date, K_{t-1} and B_{t-1}, an equilibrium from date t onwards is characterized by the path of land price, farmers' land holdings and debt, $\{(q_{t+s}, K_{t+s}, B_{t+s}) | s \geq 0\}$, satisfying equations (13), (30) and (31) at dates t, t+1, t+2, ... We continue to assume (A2), to rule out exploding bubbles in the land price.

There is a unique steady state, (q^*, K^*, B^*), with associated steady state user cost u^*, where

$$\frac{R-1}{R}q^* = u^* = \frac{\pi a - (1-\lambda)(1-R+\pi R)\phi}{\lambda\pi + (1-\lambda)(1-R+\pi R)} \tag{32a}$$

$$\frac{1}{R}G'\left(\frac{1}{m}(\bar{K} - K^*)\right) = u^* \tag{32b}$$

$$B^* = \frac{1}{R-1}(a-\phi+\lambda\phi)K^* \tag{32c}$$

Assumptions (A3) and (A4) ensure that these steady state values are positive.

To study the dynamics, it helps to substitute for the user cost from (13) into (30), so that equations (13), (30) and (31) can be expressed as a first-order nonlinear system: for $s \geq 1$:

$$q_{t+s} = Rq_{t+s-1} - Ru(K_{t+s-1}) \tag{33a}$$

$$K_{t+s} = (1-\pi)\lambda K_{t+s-1} + \frac{\pi}{\phi + u(K_{t+s})}\left[(a + \lambda\phi + q_{t+s})K_{t+s-1} - RB_{t+s-1}\right] \tag{33b}$$

$$B_{t+s} = RB_{t+s-1} + q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - \lambda K_{t+s-1}) - aK_{t+s-1} \tag{33c}$$

(33b) and (33c) also hold at date t (s = 0), given the predetermined levels of K_{t-1} and B_{t-1}.

In the Appendix we linearise (33) around the steady state and show that the characteristic equation for the eigenvalues x of the Jacobian is:

$$(x - R)(\alpha x^2 - \beta x + \gamma) = 0, \tag{34}$$

$$\text{where} \quad \alpha \equiv 1 + \frac{\theta}{\eta}(1-\lambda+\lambda\pi)$$

$$\beta \equiv 1 + \lambda R(1-\pi) + \frac{\theta}{\eta}R(1-\pi)(1-\lambda)$$

$$\gamma \equiv \lambda R(1-\pi),$$

and the parameter θ is defined as

$$\theta \equiv \frac{u^*}{\phi + u^*} = \frac{\pi a - (1-\lambda)(1-R+\pi R)\phi}{\pi(a + \lambda\phi)}. \tag{35}$$

θ is the steady state ratio of the user cost of land to the farmers' required downpayment per unit of investment. Note that $\theta < 1$, since the farmers have to finance investment in trees from their net worth. (In Section 2, θ was unity.) θ will be important to our discussion in Section 4 of the role of the noncollateralized asset.

From (34) we see that one of the eigenvalues equals $R > 1$, which corresponds to an explosive path. The product of the other two eigenvalues equals the positive constant γ/α -- which using assumption (A4) is less than 1. In the Appendix we show that these two eigenvalues will be stable and complex if and only if

$$\text{(A5)} \qquad \eta^- < \eta < \eta^+, \quad \text{where}$$

$$\eta^+, \eta^- = R(1-\pi)\theta \left(\frac{\sqrt{\lambda^2\pi + \lambda(1-\lambda)(1-R+\pi R)} \pm \sqrt{\lambda^2\pi - [1-\lambda]^2}}{1 - \lambda R(1-\pi)} \right)^2 .$$

The argument of the first square root is positive by assumption (A4); and the argument of the second square root will be positive insofar as λ is close to 1. There is little difficulty in meeting condition (A5) if π is not too close to 0 or 1. For the rest of this section, we suppose that (A5) is satisfied.

Given assumption (A2) which rules out exploding bubbles, we take the land price q_t to be a jump variable so that $(q_t, K_t, B_t)'$ lies on a two-dimensional stable manifold. For the linear approximation of (33) we show in the Appendix that this stable manifold, expressed in terms of proportional deviations from steady state, is a plane

$$\text{(36)} \qquad \hat{q}_t = \mu_K \hat{K}_t - \mu_B \hat{B}_t,$$

where $\mu_K > 0$ and $\mu_B > 0$ are functions of the parameters of the model.[30] (36) generalizes (20) from Section 2. Here $\hat{B}_t$ enters separately from $\hat{K}_t$, because in aggregate the farmers' debt repayment is no longer tied to the value of

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[30] The expressions for μ_K and μ_B are uninformative, so we do not report them in the text. See the Appendix.

their land holdings. Notice that, on the stable manifold, $\hat{q}_t$ is an increasing function of $\hat{K}_t$ and a decreasing function of $\hat{B}_t$.

Within this stable manifold, the system exhibits damped oscillations, and decays at the rate $1 - \sqrt{\gamma/\alpha}$, where γ and α are defined in (34). The intuition for why the system cycles can best be understood by reducing the dimensionality from three to two. In the Appendix we linearise (33b) and (33c) at date t (s = 0), and substitute for $\hat{q}_t$ from (36) to obtain

$$\text{(37)} \qquad \begin{pmatrix} \hat{K}_t \\ \hat{B}_t \end{pmatrix} = \begin{pmatrix} + & - \\ + & ? \end{pmatrix} \begin{pmatrix} \hat{K}_{t-1} \\ \hat{B}_{t-1} \end{pmatrix}.$$

From the sign pattern of the reduced-form transition matrix in (37), we see that our model is closely related to the classic predator-prey model discussed in the Introduction. The farmers' debt B_t plays the role of predator, and their land holdings K_t acts as prey. A rise in K_{t-1} means that farmers inherit more land at date t, which enables them to borrow more ($\partial B_t/\partial K_{t-1} > 0$). However, a rise in B_{t-1} implies that farmers have a greater debt overhang at date t, which restricts their ability to expand ($\partial K_t/\partial B_{t-1} < 0$). As the simulations in Section 5 will demonstrate, this type of system tends to exhibit not only large but also persistent oscillations when hit by a shock.[31]

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[31] While our concern here is with how the model behaves in the neighborhood of
the steady state, it should be borne in mind that predator-prey models
typically have interesting global properties, such as limit cycles. We have
not investigated the nonlinear dynamics of our model, although the
simulations we report in Section 5 are of the full nonlinear model, not the
linear approximation.

A difference between our model and a classic predator-prey model is that one
of the diagonal entries of the transition matrix in (37) may be negative: the
partial effect of B_{t-1} on B_t is ambiguous, because the direct positive effect
of rolling over debt from date t-1 may be dominated by indirect negative
effects. These indirect effects come through the negative impact an increase
in B_{t-1} has on farmers' net worth at date t -- and hence on their land demand
and the land price. (In the formula for B_t in (31), the term RB_{t-1} reflects

As in Section 2, let us consider the impact of a small, unanticipated, temporary productivity shock Δ at date t. Prior to date t, our economy is in steady state: $K_{t-1} = K^*$ and $B_{t-1} = B^*$. Now (33b) and (33c) hold at date t (s = 0) with the parameter a replaced by $a+\Delta a$. In the Appendix we linearise these two equations and appeal to equation (36) -- which is unaffected by the temporary shock at date t, because it concerns the future for which people still have perfect foresight -- to solve simultaneously for $\hat{q}_t$, $\hat{K}_t$ and $\hat{B}_t$. Here we report just $\hat{q}_t$ and $\hat{K}_t$:

$$(38) \quad \hat{q}_t = \frac{1}{\eta} \frac{\lambda\pi + (1-\lambda)(1-R+\pi R)}{1-\lambda+\lambda\pi} \frac{a}{a+\lambda\phi} \Delta$$

$$(39) \quad \hat{K}_t = \frac{1}{1+\frac{\theta}{\eta}(1-\lambda+\lambda\pi)} \left(1 + \frac{R}{R-1} \frac{\pi}{1-\lambda+\lambda\pi} \frac{\theta}{\eta} \right) [\lambda\pi + (1-\lambda)(1-R+\pi R)]^{-1} \frac{a}{a+\lambda\phi} \Delta$$

-- where θ is defined in (35). Notice that (38) and (39) generalize (21) and (22) from Section 2 to the case where $\phi > 0$ and $\pi < 1$.

Much of the discussion of the basic model in Section 2 carries over. In percentage terms, the impact on the land price, given by (38), is of the same order of magnitude as the temporary shock Δ times $\frac{a}{a+\lambda\phi}$. And the impact on the farmers' land holdings (and hence on aggregate fruit output) is large. The multiplier in (39) can be significant because of the leverage effect: a one per cent rise in land price increases the farmers' aggregate net worth by $\frac{R}{R-1} \frac{\pi}{1-\lambda+\lambda\pi} \theta$ per cent. This is not as large as in Section 2, but still can be considerably larger than unity.

As the simulations in Section 5 will show, although the impulse response to Δ may be smaller when $\pi < 1$, the effects of the shock can

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the direct effect; the indirect effects are captured by the terms $q_t(K_t-K_{t-1})$ and $\phi(K_t-\lambda K_{t-1})$.)

continue to build up after date t, before they start decaying later on --- unlike in Section 2, where they start decaying immediately. Moreover, the effects last longer: the decay rate $1 - \sqrt{\gamma/\alpha}$ is smaller when π is smaller (as long as trees are not too costly).[32]

At this point, readers who are less interested in the role of the uncollateralized asset may care to skip directly to the numerical simulations in Section 5.

4. A Special Case: The Role of the Uncollateralized Asset

The focus of Section 3 was on the dynamics of the model. In particular, we showed how introducing heterogeneity among the farmers ($\pi < 1$) uncouples their debt repayments from the value of their collateralized asset, land, and leads to cycles. We paid less attention to the role of the uncollateralized asset, trees -- beyond saying that the inclusion of trees in the farmers' production function helps rationalize (25), and brings down their debt-asset ratios to reasonable levels.[33] But ϕ has more significant effects on the dynamics of the economy, as we now see.

It is clearest to look at the special case where there is no heterogeneity among the farmers: $\pi = 1$. This rules out cycles -- Assumption (A5) does not hold -- and brings us closer to the basic model of Section 2.

The argument of Section 2 carries over to the case $\phi > 0$. Of the three

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[32] $1 - \sqrt{\gamma/\alpha}$ is smaller when π is smaller if and only if

$$a > \left(1 + (R-1)(1-\lambda)\left[\frac{1-\pi}{\pi}\right]^2 \right) (1-\lambda)\phi ,$$

which is a slightly stronger condition than (A3).

[33] The assumption that trees are specific to a farmer also helped justify why land was purchased rather than rented. See the end of footnote 13.

key equations, (11) and (13) are unchanged, and (10) becomes

$$(40) \qquad K_t = \frac{1}{\phi + q_t - \frac{1}{R}q_{t+1}} \left[(a + q_t + \lambda\phi)K_{t-1} - RB_{t-1} \right],$$

which is the special case of (30) with $\pi = 1$. Notice that ϕ appears twice in (40). The $\lambda\phi K_{t-1}$ term in the numerator is the (depreciated) value of trees inherited from date t-1, which is part of the farmers' net worth at date t. The ϕ in the denominator reflects the fact that the required downpayment per unit of land includes the cost of trees (since trees cannot be collateralized), in addition to the user cost, $q_t - \frac{1}{R}q_{t+1}$.

Consider the counterpart to (17). Suppose the economy is in steady state at date t-1. Following the unexpected temporary shock Δ at date t, the (proportional) changes in land price, $\hat{q}_t$, and the farmers' future path of land holdings, $\hat{K}_t$, $\hat{K}_{t+1}$, ..., satisfy

$$(41a) \qquad \left(1 + \frac{\theta}{\eta}\right) \hat{K}_t = \frac{a}{a + \lambda\phi}\Delta + \frac{R}{R-1}\theta\hat{q}_t \qquad \text{(date t)}$$

$$(41b) \qquad \left(1 + \frac{\theta}{\eta}\right) \hat{K}_{t+s} = \hat{K}_{t+s-1} \quad \text{for } s \geq 1 \qquad \text{(dates t+1, t+2,...)}$$

-- where, from (35), $\theta = \frac{a - (1-\lambda)\phi}{a + \lambda\phi}$, which lies between 0 and 1.

There are two kinds of difference between (41) and (17). First, the coefficients of Δ and $\hat{q}_t$ in (41a) are both smaller than in (17a): ϕ reduces the impact of both Δ and $\hat{q}_t$ on the farmers' net worth at date t, and hence on their land demand. This is because ϕ reduces leverage: the farmers' net worth includes the value of the trees inherited from date t-1.

Second, the bracketed coefficients on the left hand sides of (41) are smaller than in (17), which increases the impact of the shock on farmers' land holdings at all dates t+s, $s \geq 0$. The reason is that if the required

downpayment per unit of land comprises the user cost u_{t+} and the cost of trees, then the farmers' land demand is less sensitive to a rise in u_{t+s}. That is, ϕ reduces the choking-off effect. It is tantamount to an increase in the elasticity of the residual supply of land to the farmers from η to η/θ. We learn from (41b) that ϕ makes the changes in the farmers' future land holdings -- and hence in future user costs -- more persistent: the decay factor is $\eta/(\theta+\eta)$, compared to only $\eta/(1+\eta)$ without trees.

This additional persistence is reflected in land price. To see this, consider the counterpart to (20) for $\phi > 0$:

$$(42) \qquad \hat{q}_t = \frac{1}{\eta}\frac{R-1}{R}\sum_{s=0}^{\infty} R^{-s}\hat{K}_{t+s} = \frac{1}{\eta}\frac{R-1}{R}\frac{1}{1 - \frac{1}{R}\frac{\eta}{\theta+\eta}}\hat{K}_t.$$

(42) tells us that ϕ causes the land price to change more relative to the farmers' land holdings: without trees, the factor $\eta/(\theta+\eta)$ in the denominator reduces to $\eta/(1+\eta)$.

Altogether, then, there are a number of competing effects. To find out which one dominates, we solve (41a) and (42) for $\hat{q}_t$ and $\hat{K}_t$:

$$(43) \qquad \hat{q}_t = \frac{a}{a + \lambda\phi}\frac{1}{\eta}\Delta$$

$$(44) \qquad \hat{K}_t = \frac{1}{1 + \frac{\theta}{\eta}}\left(1 + \frac{R}{R-1}\frac{\theta}{\eta}\right)\frac{a}{a + \lambda\phi}\Delta.$$

(43) and (44) are the counterparts to (21) and (22) for $\phi > 0$. Overall, we can see that ϕ reduces the impact of the shock on land price and farmers' land holdings. In other words, the reduction in leverage is the dominant

impact effect.[34]

From our discussion of (42) we know that although ϕ may reduce the impact of a shock on both price (q_t) and quantity (K_t), it reduces the impact on quantity by more: the ratio $\hat{q}_t/\hat{K}_t$ is greater than the equivalent ratio in (21) and (22). ϕ therefore helps explain greater movement in asset prices, relative to quantities.

To sum up, there are several significant consequences of introducing an uncollateralized asset into the model. It increases the degree of persistence. It shifts the action from quantities to asset prices. And it helps bring down the farmers' debt-asset ratio to reasonable levels. The only "drawback" is that impulse responses are reduced; however, the impulse responses in Section 2 are arguably too strong anyway.

All the conclusions we have reached in this section hold for the full model of Section 3, where the farmers are heterogeneous.[35] In (38) and (39), $\hat{q}_t$ and $\hat{K}_t$ are both decreasing functions of ϕ; and $\hat{q}_t/\hat{K}_t$ increases with ϕ. Also, there is greater persistence: the decay rate $1 - \sqrt{\gamma/\alpha}$ is a decreasing function of ϕ in (34).

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[34] It is worth observing that if there were no dynamic multiplier -- i.e. if q_{t+1} were artificially pegged at q^* -- the sizes of $\hat{q}_t$ and $\hat{K}_t$ would only be

$$\hat{q}_t \Big|_{q_{t+1}=q^*} = \frac{R-1}{R}\,\frac{a}{a+\lambda\phi}\,\frac{1}{\eta}\,\Delta \qquad \text{and} \qquad \hat{K}_t \Big|_{q_{t+1}=q^*} = \frac{a}{a+\lambda\phi}\,\Delta.$$

These expressions are the counterparts to (21′) and (22′) for $\phi > 0$. Notice that, as in Section 2, the additional movement in land price attributable to the dynamic multiplier is $1/(R-1)$ times the movement due to the static multiplier. And the dynamic multiplier has a similarly large proportional effect on the farmers' land holdings.

[35] Notice that (43) and (44) specialize (38) and (39) to the case $\pi = 1$.

5. Simulations

This section presents the results of some numerical simulations. The object of this exercise is first to get a quantitative sense about how the impulse responses of the economy depend on the underlying parameters. The model has several parameters, but we choose to concentrate here on the two key ones associated with investment and dynamics: π, the arrival rate of investment opportunities; and ϕ, the cost of investing in trees. Second, by simulating the full nonlinear model of Section 3, we can confirm that the linear approximations are reasonably accurate.

Figures 3 to 5 present simulation results following a 1% unanticipated, temporary productivity shock at date 1 (i.e. Δ = 1%, the increase in the date 1 fruit output of all the farmers and gatherers). Prior to the shock, at date 0, the economy is in steady state. We use parameters that might be reasonable for a quarterly model: $R = 1.01$ (equivalent to a 4% annual real interest rate); $\lambda = .975$ (equivalent to 10% annual depreciation rate of trees); and $u(K) \equiv K - \nu$, where the intercept ν is set to make η, the elasticity of the residual supply of land to farmers, equal to 10% in steady state. Normalizing $a = 1$, we choose $\bar{K}$ so that, in steady state, the farmers use 2/3 of the total land stock. In the figures we draw q_t/q^*, K_t/K^* and B_t/B^*, the ratios of the land price, the farmers' aggregate land holding and their aggregate debt to their respective steady state values. The aggregate debt-asset ratio is defined as $B_t/[(q_t + \phi)K_t]$ for the entire farming sector; while the marginal debt-asset ratio is defined as $q_{t+1}/[R(q_t + \phi)]$, for a farmer who is investing at date t. The steady-state values of these ratios are reported at the top of each figure, denoted by "adebt" and "mdebt" respectively.

Figure 3 presents the simulation results for the case where farmers always have an opportunity to invest: $\pi = 1$. (So this is a Section 4 special case.) We set $\phi = 20$; this implies that the aggregate and marginal debt-asset ratios both equal 71%. At date 1, given the 1% temporary (i.e. one quarter) productivity increase, the land price increases by 0.43%, and the farmers' land holding and debt increase by 0.90% and 1.25%. A 0.43% increase in land price may not appear large, but it is much larger than it

Figure 3: pi = 100%, phi = 20, adebt = 71%, mdebt = 71%

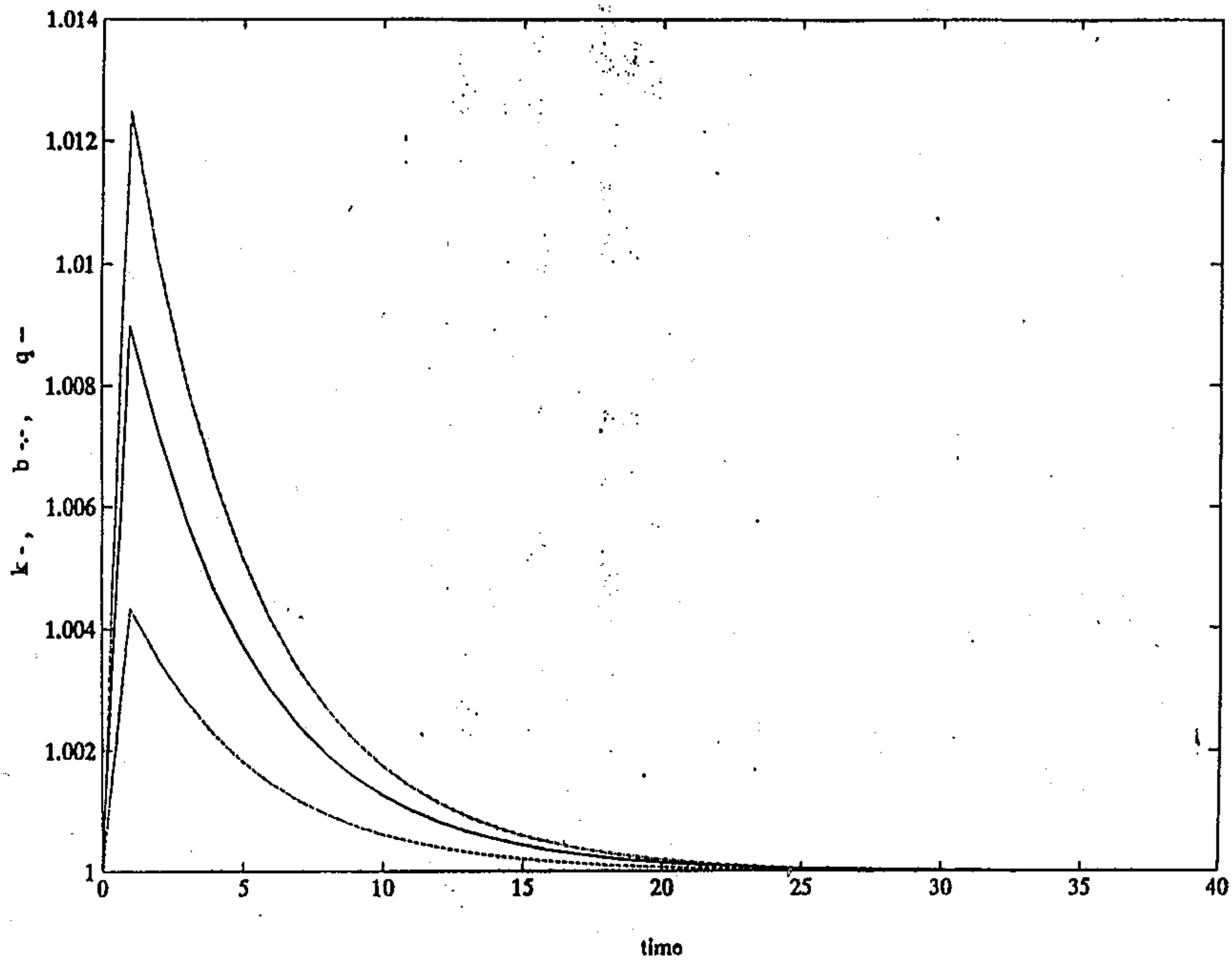
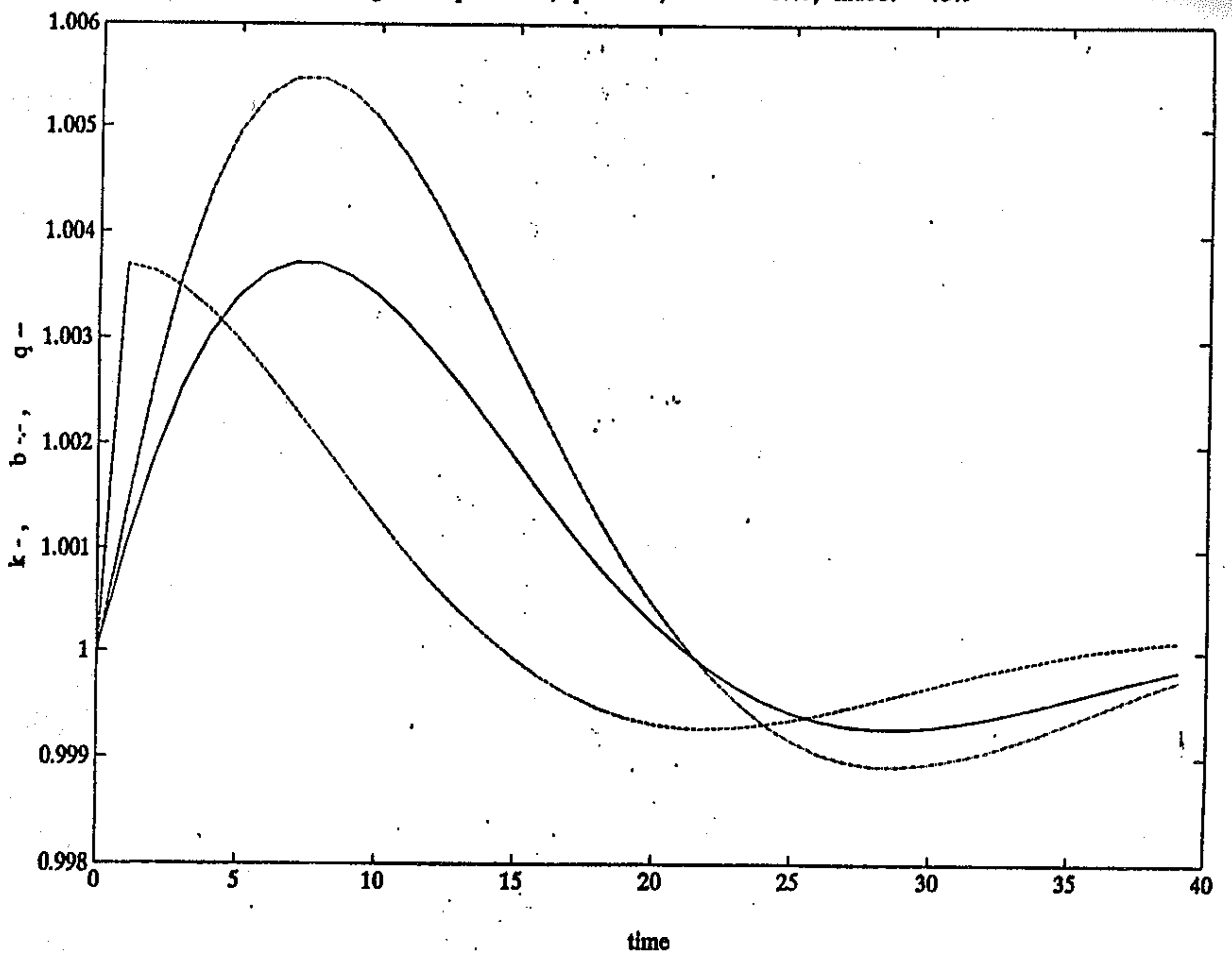


Figure 4: pi = 10%, phi = 20, adebt = 67%, mdebt = 73%



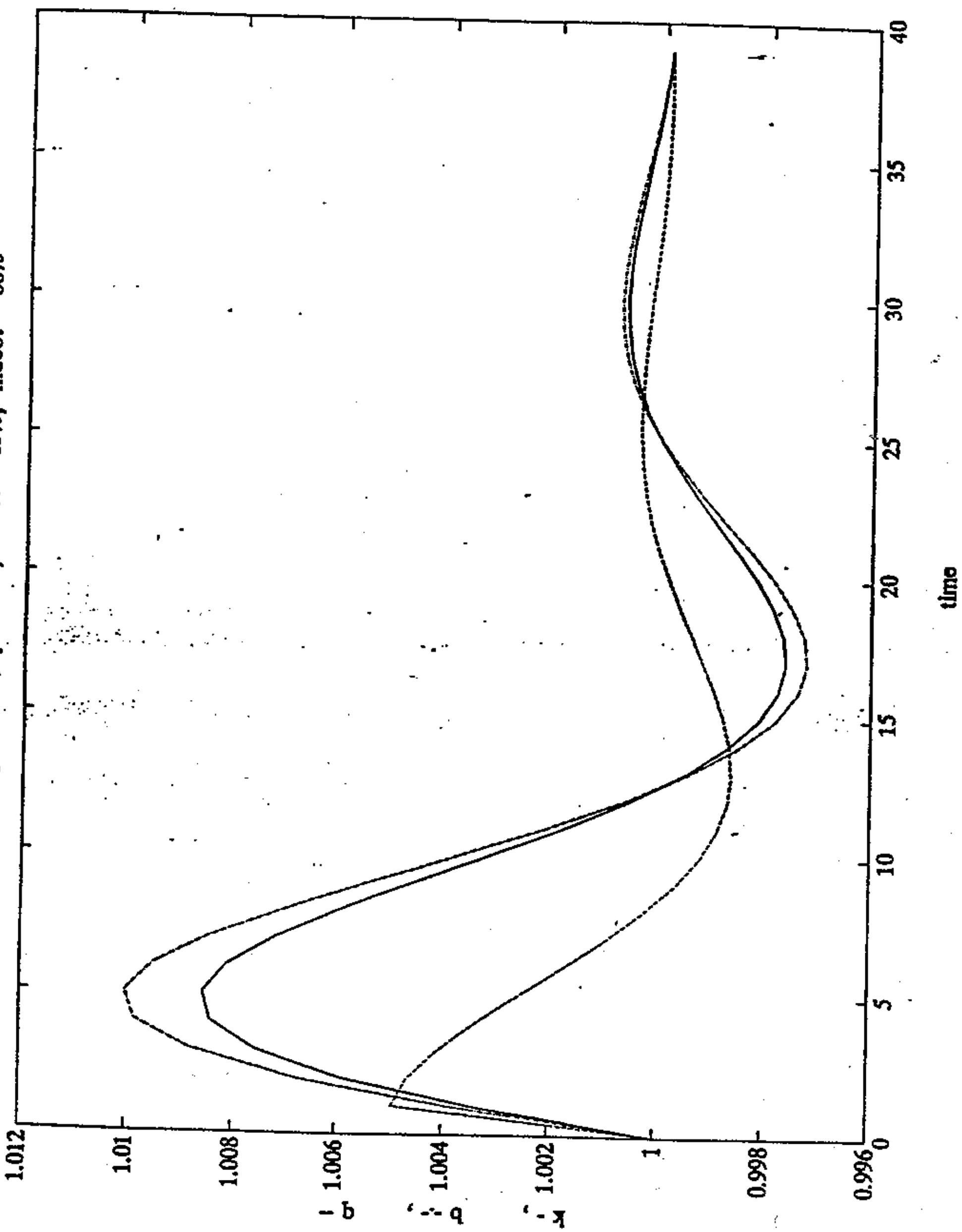


Figure 5: pi = 10%, phi = 10, adebt = 85%, mdebt = 88%

would be in a standard competitive model without credit constraints.[36] Although the residual supply of land to the farming sector is relatively inelastic (η = 10%), the effects of the shock persist (the rate of decay is 20%), because the investment cost of trees makes up a relatively large proportion of the required downpayment. Nevertheless, the effects almost disappear after 3 years (12 quarters). The movement in aggregate fruit output depends on the size of parameter c.[37] In what follows we set c = 1 (so the maximum savings rate of an individual farmer is 50%). Output is 1% higher than steady state at date 1: this is simply the direct effect of the productivity shock. At date 2 it is 0.37% higher. The sum of the all the increases in output from date 2 onwards equals 1.87%, which exceeds the direct effect at date 1.

Figure 4 presents the simulation results for the same parameters values as in Figure 3, except that the arrival rate of investment opportunities is π = 0.1 instead of 1. (So this is back to the full model of Section 3.) That is, the average interval between investments for a farmer is 2.5 years (10 quarters). Because not every farmer is investing each period, the aggregate debt-asset ratio is only 67% -- lower than the marginal debt-asset ratio of 73%. The initial impact effects of the 1% productivity shock on K_t and B_t are considerably smaller than in Figure 3 (only 0.10% and 0.13%, as compared to 0.90% and 1.25%), but they build up. After 7 quarters they peak, at 0.37% and 0.55%, before starting to cycle. The effects are more persistent, with a rate of decay of 7% (compared to 20% in Figure 3). Thanks to this persistence, the increase in q_t at date 1 is not much smaller than in Figure 3: 0.37% rather than 0.43%. Thus the price movement becomes more volatile relative to quantity if not all the farmers invest simultaneously. The length of the cycle is about 10 years, and the land price leads output by

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[36] Recall that in a standard competitive model, the date 1 land price would not increase at all, because the shock does not affect the future. Alternatively, one might consider the possibility that, although the shock is announced at date 1, it will not happen until date 2 -- in which case the date 1 land price would increase only in the order of the net real interest rate, 0.01%.

[37] The parameter c has no effect on the dynamics of q_t, K_t and B_t as long as it satisfies condition (A1').

about 7 quarters. The sum of the increases in output between date 2 and the midpoint of the cycle (date 22) is 1.79%, which again exceeds the direct effect (1%) at date 1. The sum of the decreases in output over the second half of this cycle is 0.35%.

Figure 5 presents the simulation results for the same parameters values as in Figure 4, except that the trees are less costly: $\phi = 10$ rather than 20. The debt-asset ratios are higher (85% in aggregate, and 88% for the marginal farmer). Compared to Figure 4, we see that the effects of the shock are initially larger, but are less persistent. Also, the price movement becomes less volatile relative to quantity as the noncollateralized asset becomes less important. These findings confirm the conclusions we reached in Section 4 about the role of the uncollateralized asset. The length of cycle in Figure 5 is shorter than in Figure 4: 6 years rather than 10. The sum of the increases of output during the first half of the cycle, starting at date 2, is 1.97%; while the sum of the decreases during the second half is 0.57%.

Let us now check the accuracy of our earlier linear approximations. From those approximations, we would have predicted that the land price, the farmers' land holding and debt would respectively increase by 0.49%, 1.0% and 1.4% for the parameters of Figure 3, rather than the true amounts 0.43%, 0.9% and 1.25%. Thus the linear approximation introduces an upward bias. For the parameters of Figure 4, the bias is smaller: we would have predicted corresponding increases of 0.40%, 0.11% and 0.14%, compared with the true figures 0.37%, 0.10% and 0.13%.

6. Spillovers

As the model is constructed, there cannot be any positive spillovers between the farming and gathering sectors, since their combined land usage must always sum to $\overline{K}$. In order to study spillover effects, we extend the basic model of Section 2, to have two farming sectors, 1 and 2.

Suppose there are different types of fruit. Gatherers make "regular" fruit, with the same production function as before. Farmers, however,

produce slightly differentiated fruit. The farming technology is very similar to that in Section 2. In sector $i = 1$ or 2, a farmer with land k_{it-1} at date t-1 produces $a_i k_{it-1}$ tradeable fruit at date t, together with $c_i k_{t-1}$ nontradeable fruit. The only difference is that the tradeable fruit is peculiar to that sector. The nontradeable fruit is equivalent to regular fruit in consumption value to the farmer.

We assume that consuming a bundle comprising x_{1t} fruit from sector 1 and x_{2t} fruit from sector 2 is equivalent to consuming x_t regular fruit, where

$$(45)\qquad x_t = \left(x_{1t}^{1-\varepsilon} + x_{2t}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

The parameter $\varepsilon > 0$ is the inverse of the elasticity of substitution in consumption between the two types of fruit. We take ε to be small: any positive value will pin down the size of each farming sector in equilibrium, and ensure that neither sector disappears.[38]

Let regular fruit be the numeraire good. Then at date t, the competitive price, p_{it} say, of fruit from farming sector i is equal to the marginal rate of substitution:

$$(46)\qquad p_{it} = (a_i K_{it-1})^{-\varepsilon} \left((a_1 K_{1t-1})^{1-\varepsilon} + (a_2 K_{2t-1})^{1-\varepsilon} \right)^{\frac{\varepsilon}{1-\varepsilon}} \qquad \text{for } i = 1,2$$

-- where K_{it-1} denotes the aggregate land holdings of the farmers in sector i at date t-1.

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[38] If the products of the two farming sectors were perfect substitutes ($\varepsilon = 0$) then the sector with the higher productivity would eventually take over the whole market -- unless $a_1 = a_2$, in which case the sizes of the sectors would be indeterminate.

As in (10), the aggregate land holdings of the farmers in sector i at date t is given by

$$(47) \qquad K_{it} = \frac{1}{q_t - \frac{1}{R}q_{t+1}} \left((a_i p_{it} + q_t)K_{it-1} - RB_{it-1} \right) \qquad \text{for } i = 1,2,$$

where B_{it-1} denotes their aggregate debt at date t-1. (The only substantive difference between (47) and (10) is that the tradeable fruit output $a_i K_{it-1}$ is priced at p_{it} rather than at unity.) And as in (11), the aggregate debt of the farmers in sector i at date t is given by

$$(48) \qquad B_{it} = \frac{1}{R}q_{t+1}K_{it} \qquad \text{for } i = 1,2.$$

The land market equilibrium condition is the same as (13), except that the farmers' land holdings from the two sectors are added together:

$$(49) \qquad q_t - \frac{1}{R}q_{t+1} = u(K_{1t} + K_{2t}).$$

For given levels of K_{it-1} and B_{it-1}, i = 1,2, an equilibrium from date t onwards is characterized by a sequence $\{(q_{t+s}, p_{it+s}, K_{it+s}, B_{it+s}) | s \geq 0, i = 1,2\}$ satisfying equations (46), (47), (48) and (49) at dates t, t+1, t+2, ...[39]

[39] We continue to assume (A2), and a suitably modified version of (A1):

$$c_i > 2^{\frac{\varepsilon}{1-\varepsilon}} (R-1)a_i \qquad \text{for } i = 1,2.$$

Let us consider the impulse response to a sector-specific technology shock. Suppose the economy is in steady state at date t-1, and, for the purpose of comparison, suppose $a_1 = a_2 = a$. As the two farming sectors are symmetric, the steady state is described by (14) with $K_1^* = K_2^* = \frac{1}{2}K^*$ and $B_1^* = B_2^* = \frac{1}{2}B^*$. At the start of date t there is an unanticipated, temporary increase in the output of Sector 1 only: the harvest of the farmers in sector 1 is $1 + \Delta$ times higher than expected.

We can follow the argument of Section 2 to show that, for a small shock Δ, the proportional changes in the date t land price and farmers' land holdings are approximately given by

$$(50) \qquad \hat{q}_t = \frac{1}{2}\frac{1}{\eta}\Delta$$

$$(51a) \qquad \hat{K}_{1t} = \left(1 + \frac{1}{2(R-1)(1+\eta)} - \frac{\varepsilon}{2} \right) \Delta$$

$$(51b) \qquad \hat{K}_{2t} = \left(\frac{1}{2(R-1)(1+\eta)} + \frac{\varepsilon}{2} \right) \Delta.$$

Comparing (50) with (21), we see that the effect of the sector-specific shock on q_t at date t is exactly the half that of the aggregate shock -- simply because only half of the farmers experience the specific shock. And comparing (51) with (22), we see that the proportional change in $K_{1t} + K_{2t}$ -- i.e. the average of (51a) and (51b) -- equals half that of K_t in (22).

More interesting is to enquire how this shock is divided across the

(The factor $2^{\frac{\varepsilon}{1-\varepsilon}}$ here is the steady state value of p_{it}, in the symmetric case $a_1 = a_2$.)

sectors. The first term inside the large bracket in (51a) is the direct impact of the productivity shock on the farmers in sector 1. However, given R close to 1, this first term is dwarfed by the second term, the indirect effect on the farmers' land demand arising from the change in their net worth caused by the jump in land price.[40] But this indirect benefit is enjoyed by the farmers in the other sector: (51b) is almost as large as (51a). That is, because all the farmers hold land, the size of the immediate spillover is significant.

Thereafter, the changes in the farmers' land holdings in each of the sectors follow the two-sector analogue of (18): for $s \geq 1$,

$$(52)\qquad \begin{pmatrix} \hat{K}_{1t+s} \\ \hat{K}_{2t+s} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2(1+\eta)} - \frac{\varepsilon}{2} & -\frac{1}{2(1+\eta)} + \frac{\varepsilon}{2} \\ -\frac{1}{2(1+\eta)} + \frac{\varepsilon}{2} & 1 - \frac{1}{2(1+\eta)} - \frac{\varepsilon}{2} \end{pmatrix}^{s} \begin{pmatrix} \hat{K}_{1t} \\ \hat{K}_{2t} \end{pmatrix}.$$

The off-diagonal entries of the transition matrix in (52) are spillover effects. The first term, $-\frac{1}{2(1+\eta)}$, reflects crowding-out: greater production in one sector crowds out production in the other, through an increase in the user cost of land. The second term, $\frac{\varepsilon}{2}$, reflects demand linkage: higher demand in one sector stimulates demand in the other through changes in the relative price. For $\varepsilon < \frac{1}{1+\eta}$, the crowding-out will dominate the demand linkage. The positive diagonal entries in the transition matrix capture the fact that an increase in the land holdings of the farmers in sector i at date t+s-1 increases their net worth, and hence their land demand, at date t+s. The implication of (52) is that the initial increases $\hat{K}_{1t}$ and $\hat{K}_{2t}$ persist;

[40]The $\frac{\varepsilon}{2}$ term in (51) represents demand linkage: the expansion in sector 1 is partially offset by a fall in their product price p_{1t}, which boosts sector 2's demand and product price p_{2t}. These terms are negligable for small ε.

and the two sectors co-move after a shock, at least for a time.[41]

In sum, we have shown how production in the different farming sectors tends to move together, through the common effects of land price on the farmers' net worth -- even if the underlying shocks are sector-specific.

7. Final Remarks

In the paper we constructed a model of a dynamic economy which, at the aggregate level, is deterministic; and we then hit the economy with an unexpected temporary shock. Although this approach succeeds in keeping the analysis tractable, it skirts around some central issues. The next step in the research is to construct a fully-fledged stochastic model, in which a shock is not a zero probability event and is rationally anticipated.

The key question is: to what extent can contingent debt contracts be written? There are a number of explanations for why it may be impossible to condition debt repayments on idiosyncratic shocks. For example, such shocks may not be observable to outsiders, like the courts; see Hart and Moore (1989).[42] However, it is less clear why the terms of a contract cannot be made sensitive to aggregate events, such as movements in the price of land or the interest rate. This is a difficult matter to resolve.

Let us turn to less thorny issues. A weakness of our model is that it provides no analysis of who becomes credit constrained, and when. We merely rely on the assumption that different agents have different technologies: some, the farmers, have a (profitable) linear technology and face credit limits; whereas others, the gatherers, have a concave technology and are

[41]The eigenvalues of the transition matrix in (52) are $\frac{\eta}{1+\eta}$ and $1-\varepsilon$, which both lie between 0 and 1.

[42]Even if financial contracts can be written contingent on idiosyncratic shocks, it may not be efficient to diversify risk fully, for standard moral hazard reasons.

unconstrained. However, one can instead assume that agents differ in their levels of accumulated wealth. Ortalo-Magné (1994) constructs an overlapping generations model of farming and agricultural land prices, in which agents live for many periods. Farmers borrow against their land holdings, and are most likely to face borrowing constraints when they are young and relatively poor. As they save and accumulate assets, they become unconstrained.[43]

Since the savings decision plays an important role in this class of overlapping generations model, it is unsatisfactory to assume (as we did in the paper) that preferences are linear and that there is a technological upper bound on the savings rate. There is no conceptual problem with assuming concave preferences, but the model becomes analytically much harder. One great advantage of doing so, however, is that one can then investigate how the interest rate interacts with asset prices, output, and the distribution of wealth.

Finally, it would be interesting to relax the assumption that, on the supply side, the credit market is anonymous. In the paper we have implicitly taken the position that debt contracts can be freely traded by creditors -- because the value of a debt contract equals the value of the collateral, land, which is priced in a market. However, the identity of the creditor may matter. A particular creditor may have additional information about, or leverage over, a particular borrower, which enables them to lend more. Such debt contracts are unlikely to be tradeable at full value. Once anonymity is dropped, the net worth of creditors, and the value of their collateral, starts to matter. The interaction between asset markets and credit markets which we have highlighted in this paper will be even richer if both sides of the credit market are affected by changes in the price of their collateralized assets.

[43] Firms may well go through similar kinds of life cycle. An alternative to the overlapping generations framework would be to asssume that agents face a stochastic technology, as in Scheinkman and Weiss (1986).

Appendix

This appendix studies a linearized version of the model in Section 3, in order to examine the dynamics. Consider the linear approximation of (33), expressed in terms of proportional deviations from the steady state:

$$\text{(a1)} \qquad \begin{pmatrix} \hat{q}_{t+s} \\ \hat{K}_{t+s} \\ \hat{B}_{t+s} \end{pmatrix} = J \begin{pmatrix} \hat{q}_{t+s-1} \\ \hat{K}_{t+s-1} \\ \hat{B}_{t+s-1} \end{pmatrix}$$

where the J is the Jacobian (J_{ij}), $i,j = q,K,B$, in elasticity form. (For example, J_{KB} is the derivative of K_{t+s} with respect to B_{t+s-1}, multiplied by B^*/K^*.) From (33), J is given by

$$\text{(a2)} \quad J = \begin{bmatrix} R & -\frac{1}{\eta}(R-1) & 0 \\ J_{Kq} & J_{KK} & J_{KB} \\ \frac{K^*}{B^*}(\phi+q^*)J_{Kq} & \frac{K^*}{B^*}\left[(\phi+q^*)J_{KK}-(a+\lambda\phi+q^*)\right] & \frac{K^*}{B^*}(\phi+q^*)J_{KB} + R \end{bmatrix}$$

where

$$J_{Kq} = \frac{1}{\alpha}\frac{\pi\theta R^2}{R-1}, \qquad J_{KK} = \frac{1}{\alpha}\left(\lambda(1-\pi) + \pi\,\frac{a+\lambda\phi+q^*}{u^*+\phi} - \pi R\,\frac{\theta}{\eta} \right), \qquad J_{KB} = -\frac{\pi R}{\alpha(u^*+\phi)}\frac{B^*}{K^*},$$

$$\eta = \frac{u(K^*)}{K^* u'(K^*)}, \qquad \theta \equiv \frac{u^*}{u^*+\phi}, \qquad \text{and} \qquad \alpha \equiv 1 + \frac{\theta}{\eta}(1-\lambda+\lambda\pi).$$

The eigenvalues x of J solve the characteristic equation (a3) $$\det(J-xI) = 0,$$

where I is the 3x3 identity matrix. (a3) leads to equation (34). Thus one of the eigenvalues of J equals R. The other two eigenvalues are complex if and only if

(a4) $$\beta^2 - 4\alpha\gamma < 0.$$

Using the expressions for α, β and γ given in (34), (a4) is equivalent to condition (A5). We assume (A5) is satisfied what follows. (In the text these eigenvalues are shown to be stable.)

Given Assumption (A2), we take the land price q_t to be a jump variable so that $(q_t, K_t, B_t)'$ lies on a two-dimensional stable manifold. For the linear approximation (a1), the stable manifold is the plane spanned by the two eigenvectors, $\underset{\sim}{e}$ and $\underset{\sim}{e}^*$ say, corresponding to the two stable complex eigenvalues of J, x and x*. Solving for $J\underset{\sim}{e} = x\underset{\sim}{e}$, one eigenvector equals (up to a scalar)

(a5) $$\underset{\sim}{e} = \left(1, \quad \frac{R-x}{R-1}\eta, \quad -\frac{1}{J_{KB}}\left[J_{Kq} + \frac{R-x}{R-1}\eta(J_{KK}-x) \right] \right)';$$

and the other, $\underset{\sim}{e}^*$, is the complex conjugate of $\underset{\sim}{e}$. The two-dimensional stable manifold satisfies:

(a6) $$\begin{vmatrix} \hat{q}_t & & \\ \hat{K}_t & \underset{\sim}{e} & \underset{\sim}{e}^* \\ \hat{B}_t & & \end{vmatrix} = 0.$$

Solving (a6) for $\hat{q}$, we get equation (36) in the text, where

$$\mu_K = \frac{\pi}{\eta(1-\lambda+\lambda\pi)(u^*+\phi)}(q^*+\phi) \quad \text{and} \quad \mu_B = \frac{\pi}{\eta(1-\lambda+\lambda\pi)(u^*+\phi)}\frac{B^*}{K^*}$$

The reduced form transition matrix in (37) is

(a7) $$\begin{pmatrix} \hat{K}_t \\ \hat{B}_t \end{pmatrix} = \begin{pmatrix} D_{KK} & D_{KB} \\ D_{BK} & D_{BB} \end{pmatrix} \begin{pmatrix} \hat{K}_{t-1} \\ \hat{B}_{t-1} \end{pmatrix}$$

where

$$D_{KK} \equiv J_{KK} + J_{Kq}\mu_K = \frac{1}{\alpha}\left((1-\pi)\lambda + \pi\frac{a+\lambda\phi+q^*}{u^*+\phi}\left[1 + \frac{R}{R-1}\frac{\pi}{1-\lambda+\lambda\pi}\frac{\theta}{\eta}\right] \right)$$

$$D_{KB} \equiv J_{KB} + J_{Kq}\mu_B = -\frac{1}{\alpha}\frac{B^*}{K^*}\frac{RN}{a+\lambda\phi}\left(1 + \frac{R}{R-1}\frac{\pi}{1-\lambda+\lambda\pi}\frac{\theta}{\eta} \right)$$

$$D_{BK} \equiv J_{BK} + J_{Bq}\mu_K = \frac{1}{\alpha}\left(1-R+\pi R + \frac{\theta\pi R}{R-1} + N\frac{\theta}{\eta}\left[\frac{1}{a+\lambda\phi-\phi} + \frac{R}{R-1}\frac{\pi}{1-\lambda+\lambda\pi}\frac{1}{a+\lambda\phi} \right] \right)$$

$$D_{BB} \equiv J_{BB} + J_{Bq}\mu_B = -\frac{1}{\alpha}\frac{B^*}{K^*}\frac{R}{a+\lambda\phi}\left(1-R+\pi R - \frac{\lambda(1-\pi)(R-1)\phi}{a+\lambda\phi-\phi} + N\frac{\theta}{\eta}\left[\frac{a+\lambda\phi}{a+\lambda\phi-\phi} + \frac{R}{R-1}\frac{\pi}{1-\lambda+\lambda\pi} \right] \right)$$

and where $N \equiv \lambda\pi + (1-\lambda)(1-R+\pi R) > 0$. Note that D_{KK} and D_{BK} are both positive, and D_{KB} is negative. The sign of D_{BB} is ambiguous.

In order to compute the impulse response to the temporary productivity shock Δ, we use the linear approximation of equations (33b) and (33c):

(a8) $$\alpha \hat{K}_t = \pi \frac{q^*}{u^*+\phi} \hat{q}_t + \pi \frac{a}{u^*+\phi} \Delta$$

(a9) $$\hat{B}_t = (q^*+\phi) \frac{K^*}{B^*} \hat{K}_t - a \frac{K^*}{B^*} \Delta.$$

Solving (a8), (a9) and (36) for $\hat{q}_t$, $\hat{K}_t$ and $\hat{B}_t$, we obtain (38), (39) and

(a10) $$\hat{B}_t = \frac{1}{\alpha} \frac{a}{a+\lambda\phi} \left(1-R+\pi R - \frac{\lambda(1-\pi)(R-1)\phi}{a+\lambda\phi-\phi} + N \frac{\theta}{\eta} \left[\frac{a+\lambda\phi}{a+\lambda\phi-\phi} + \frac{R}{R-1} \frac{\pi}{1-\lambda+\lambda\pi} \right] \right) \Delta.$$

At date t, a productivity shock Δ has the same impact on the farmers' net worth as a reduction in their debt obligations by a proportion $\frac{aK^*}{RB^*} \Delta$, and so has the same impact on production and allocation (aside from the change in output at date t itself). To confirm this, note that $\hat{K}_t$ and $\hat{B}_t$ in equations (39) and (a10) can be written

(a11) $$\hat{K}_t = D_{KB} \left(- \frac{aK^*}{RB^*} \Delta \right)$$

(a12) $$\hat{B}_t = D_{BB} \left(- \frac{aK^*}{RB^*} \Delta \right)$$

where D_{KB} and D_{BB} are coefficients of the reduced form transition matrix in (a7).

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